ON THE LOGIC OF MEASUREMENT

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INTRODUCTION

Distinctions which are the outcome of special investigations and which have well defined and adaptable meanings in the contexts wherein they operate successfully, often take on a rigidity and a mutually incompatible exclusiveness when they are elaborated in abstraction from those contexts. When subject matter is approached with these elaborated distinctions, the existence of both the subject matter whose distinctions they presumably were, and of the investigations which had used them successfully, not seldom turn out to be irresolvable problems. The history of thought, as a consequence, is replete with attempts to lay the universe on the procrustean bed of one of several elements of analysis. That nature is a unitary principle, or a disconnected plurality of events; an interconnected, unchanging whole, temporally complete, or a flux of evanescent qualities, forever incomplete; that nature is all form, rationally determinate, or that nature is the hap-hazard play of blind, material energies, are a few of the more obvious themes upon which speculative thought has played its variations. Nevertheless the striking resolutions of problems of this genre have usually pointed to the falseness of the antitheses and of the supposed incompatibilities of the disjoined terms. Justice is rendered, both to the complexity of subject matter and to the inelligible connection of its distinguished parts, by mitigating the oppositions of dialectic disonances. Justice is rendered through the rediscovery that the distinctions are after all *distinctions,* which become meaningless without other characters from which they may be distinct, and outside of the contexts wherein they were drawn.

The achievements of the natural sciences have been often a platform from which to project ambitious philosophic surveys of nature. But the inferences which have been usually drawn from those achievements, have made it difficult for many thinkers to admit a discoverable *order* in nature without surrendering the claims of qualitative events to a place *in* nature. The distinctions in subject matter which the sciences have been compelled to recognize and employ, have been converted into an exhaustive definition of "nature," so that the existence in "nature" of other characters than these has been repeatedly denied. *Otherness,* in Professor Sheldon's words, has been taken as equivalent to *opposition* and *denial.*[1] The distinction between those characters of the world measurable as the scientist measures them, and those characters not so measurable, has served to produce a metaphysical chasm between a supposedly ontological order of relations and a subjective domain of qualities, or between a realm of essence intuitively clear and knowable and a realm of matter reached only by an animal impulse, rationally unjustifiable.

The present essay is a modest attempt to study this distinction on its home ground and in a small sector of it. The center of interest is located in the process of measurement used in physics; for the essay has developed from the conviction that even a partial exploration of that process whereby abstract symbols are translated into existential traits should have some illuminating consequences for a philosophy of nature.

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[1] *Strife of Systems and Productive Duality,* pp. 470 ff.

It is obviously unwise to attempt an exhaustive definition of nature in terms of any set of specific characters, if one wishes to retain for nature whatever events, characters, or relations any inquiry may discover. Consequently, by nature will be understood the totality of events (if there is a totality,) perceived or inferred, possessed or capable of being possessed, known or to be known, together with the order of their coexistence and succession.

It follows, however, that since "nature" and "natural" are not differentiating, but rather denotative terms, very little seems to be affirmed in declaring oneself a naturalist in science and philosophy. But an important difference in the meaning of "nature" and "naturalism" can be detected.

The content of what is designated by "nature," as is well known, does not remain invariable with the progress of inquiry. Nature may mean the atoms and the void, the evolution of the organic from the lifeless, the embodiment of forms through the ceaseless activity of a tireless matter. Consequently, philosophies which profess equally their naturalism, may yet be poles apart in the characters they select as generic traits of existence. Naturalists have been identifiable, not so much by their unanimity in affirming the irreducible characters of nature, but by their common conviction that reflective inquiry may discover the order of the birth and decay of things; that thought and ideal forms have physical efficacy only in virtue of the operations of matter of which they are expressions; and that the ideals of life must be based upon the satisfactions achievable in a material world.

Naturalism indicates, therefore, a *temper* and a *method* as well as a metaphysics. For nature, as systems of events, may conceivably be such as the most fanciful poet has imagined her to be; nature may be the domain for the operations of demons, fairies, disembodied spirits, and physically efficacious ideal forms. But a naturalist will be one who will find himself at home in nature whatever her constitution may be. He will find himself at home there, however, because he has no other home to go to. It is a readiness to learn from nature herself the manner of her operations, that characterizes his temper and his method.

Consequently, although man's most successful enterprises will teach him the principles in terms of which the rest of nature may be read, he can retain his naturalism only in so far as his principles are continually resubmitted to verification. He will remain a naturalist only at the price of understanding that the distinctions he makes and the characters of nature he selects for attention, have not *therefore* acquired a cosmic importance. He will retain his naturalism only when he acknowledges that the highest flights of science as well as the most profound works of art have their basis in the needs of his body and in its sensitivity to the influences of his environment. The naturalistic temper in science and philosophy will reveal itself in the critical surveillance of one's intellectual instruments to disclose the needs from which they arise and to verify the adequacy of their resolving power.

It is upon the basis of such a naturalism that this essay is projected.

I wish to express my grateful acknowledgement for the assistance I have received in the preparation of this essay. Professor Morris R. Cohen has been to many of his students the "father Parmenides" upon whom indeed they may

have to "lay their hands" occasionally, but who has always been their splendid and shining guide to philosophy as a rational discipline. I cannot adequately acknowledge what I owe to his teaching and conversation, his encouraging help and kindly interest. In the writings of Professor Dewey I have found not only a basis for whatever philosophy I can also call my own, but the courage to break through some of the dialectic mires which beset the understanding. To Professor Woodbridge I am overwhelmingly indebted for the inestimable stimulus of his clarity of vision, for the generosity of his encouragement and confidence. If I have not indicated my obligations to him and to Professor Dewey in greater detail in the pages of this essay, it is only because the influence of their thought is too clearly upon them. To the sustaining friendship of Dr. Sidney Hook I owe more than I can say. His encouragement and his contagious vitality have given me the necessary strength to work my way through periods of great stress. To Dr. Michael Kraus and Dr. Joseph Ratner I am grateful for their helpfulness when sympathetic understanding was most needed.

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CHAPTER I

QUALITY

"Then by the bed-side, where the faded moon
Made a dim, silver twilight, soft he set
A table, and half anguish'd, threw thereon
A cloth of woven crimson, gold, and jet:—
O for some drowsy Morphean amulet!
The boisterous, midnight, festive clarion,
The kettle-drum, and far-heard clarinet,
Affray his ears, though but in dying tone:
The hall door shuts again, and all the noise is gone.

And still she slept an azure-lidded sleep,
In blanched linen, smooth, and lavender'd,
While he from forth the closet brought a heap
Of candied apple, quince, and plum, and gourd;
With jellies soother than the creamy curd,
And lucent syrops, tinct with cinnamon;
Manna and dates, in argosy transferred
From Fez; and spiced dainties, every one
From silken Samarcand to cedar'd Lebanon."

Eve of St. Agnes.

"By use there is sweet, by use there is bitter; by use there is warm and by use there is cold; by use there is color. But in sooth there are atoms and the void."

Democritus.

The striking contrast which has been noted by Eddington between the wind and waves of the poet's ecstasy and the wind and waves of the equations of hydrodynamics is a theme capable of many variations and much illustration. It is perhaps the most impressive single theme to which the thoughts of reflective men repeatedly return and with which they wrestle. For the "intense, clear, star-sown vault of heaven," which often helps to maintain the stoic's proud humility, is not very obviously the heaven studied either by statistical astronomy analyzing the distribution of the stars, or by the mathematician engrossed in the n-body problem. And the colors of dead leaves, "yellow, and black, and pale, and hectic red" of which Shelley's ode sings, are not easily identified with the vibration frequencies and energy levels which are the explicit subject matter of atomic physics. The world of our common-day experience is very remote indeed from the latest "explanations" of it by the natural sciences, and that contrast between the naive and sophisticated approach receives increased emphasis with the progress of mathematical physics. For the world permeated by sensory and affective qualities, which is the object of immediate experience, is transformed more completely by that progress into a world where qualities seem superfluous because inevitably absent, like slain opponents at their victor's banquet.

The importance, if not the significance, of this transformation has been

often noted. And thinkers have not been wanting who have interpreted the contrast indicated above, as the contrast between appearance and reality, illusion and truth, shadow and substance; either half of the antithesis being identified in some cases with the qualitative manifold, in others with the relational structure. Unfortunately, categories like appearance and reality indicate much more the ethical and esthetic preferences of men than the nature of the connection between the common-day world and that other world so frequently discovered by reflective analysis. Even appearance has a character of its own; and one who inquires into the relation between substance and shadow is not helped at all by being offered as a solution what is, in fact, the problem itself. If it be claimed that only the realm of forms is real, the connection of the flux with those forms is not exhibited in the metaphor of time as the moving image of eternity; if the real size of a tower is not its apparent size, *that* distinction is not clarified by the geometry of projection for which every size is equally real and apparent.

It is the thesis of this essay, and a very naive thesis it perhaps is, that the contrast so often referred to is best interpreted by saying that the abstract world of science is the *formulation,* the intellectual *expression,* of certain selected characters pervasive in the immediate qualitative world of daily experience. One could say this also by declaring that science is the truth of those characters, were not the truth so exclusively associated with the real—as if the truth of existence could be more real than existence itself. To maintain this thesis, the process of that formulation as it occurs in the sciences must be indicated. What is the connection between the logico-mathematical structures and the qualities which physics, for example, seems to neglect? It is suggested here that that neglect is only apparent: there are no sensory qualities explicitly present, for example, in the Hamiltonian canonical equations of dynamics. But that neglect is apparent only *within* the strict dialectic of the formulation, and vanishes when the question of the existential meaning of that dialectic is raised: the variables of those equations receive a sensory interpretation when the flight of a projectile is calculated by their means. The reference of those structures to something not exclusively structural can be exhibited by tracing the transition from quality, immediately possessed to quantity, which is the intellectual elaboration of its relational aspects.

The actual performance of such a task is hardly one for a single individual; but in lieu of the performance, the philosophy implicit in the task may be made the guide in a survey of some limited portion of physics. Consequently, certain extremes must be avoided. Positivists like Mach who are enamored of the immediately given, but are strangely oblivious of the fact that all discourse involves a transcendence to what is presented, deny ontological status to relational factors in subject matter. But more recent psychology, like *Gestalt,* should teach us that just such relational characters are discoverable in immediate experience, even though their emergence into a cognitive status requires the activity of minds and bodies. It is bad observation, not the nature of reality, which deprives relations of that immediacy which, for positivists, defines the real. Furthermore, if transcendence of what is immediate is ubiquitous in reflection, no metaphysical *problem* should be made of it.[1] The problems raised

[1]As it is, e.g., in N. Hartmann, *Metaphysik der Erkenntnis,* p. 61 ff.

should be empirical. They should be challenges to point to evidence which justifies particular instances of implication and extension of meaning. Finally, attempts to assign to relational structure an exclusive ontological rank, as in Cassirer, who like all neo-Kantians dissolves specificity into a relational construction, fail in explaining the undeniable relevance of that structure to the prognostic activity of empirical science in the familiar qualitative world. The plan of the first two chapters of this essay is accordingly, (1) to discuss briefly the status and function of quality, and (2) to defend their ontological character, in the present chapter; and (3) to describe the occasion and method of quantitative procedure in the next.

I

To attribute priority to a subject matter or character over other subject matters or characters, without specifying the sense in which the predication is made, is not very illuminating. How shall one understand the assertion that general qualities are prior to particular things; or that the study of logic is prior to the study of nature; or that essence is prior to existence; or that matter is prior to mind; or that the state is prior to the individual? Without further specification, not only is the truth-value of such propositions undetermined, but their very meaning as well. At least four among other senses of priority may be distinguished. (1) Some characters or subject matters are prior to others in the sense of being more familiar or better known. So the language of one's own country is generally prior to a foreign language. (2) They may be prior to others in time or existence, as Socrates is prior to Aristotle. (3) They may be prior in concreteness, in the sense that any subject matter is prior to the distinctions which are its own. So a curve is prior to its convexity. (4) They may be prior in universality, so that A is prior to B, if B is a case of A. In this sense a theorem about a polygon is prior to a theorem about a triangle.

Is there any sense in which priority may be predicated of quality? (*a*) In at least the first sense, quality may claim priority over the relational structures which science discloses. It is in a world indubitably qualitative that our births, joys, sorrows, and deaths take place. It is in this world that careers occur, choices are made, poetry is written, paintings and symphonies composed, friendships formed, and men and nature worshipped. The quantitative treatment of qualities intuitively felt and enjoyed, and the painful elaboration of the implications of that procedure, are very infrequent and remote adventures. To some, indeed, they are very barren and joyless adventures of the human spirit. It is a testimony to the frequency and emotional intensity of the primacy of quality in this sense, that a poet could say: *"Am farbigen Abglantz haben wir das Leben."*

Nevertheless, however distant the world of the physicist may seem to be from the work-a-day world of everyman, the transformation which our ideas of the latter world must undergo to become the former, is in principle little different from the transformations that occur daily on a less sophisticated level. For whenever, instead of being possessed in their uncertain tenure, qualities are *talked about,* we are on the way to relational and quantitative structures. Whenever the twinkle in a friend's eye is taken as a sign for the irony not

apparent in his words, or whenever the explicit contents of a letter are used for inferring the hidden intent of its writer, the principles of the logic which the physicist invokes are already invoked here. The description of the sunset is not the sunset. But sunsets may be described because what-it-is-to-be-a-sunset, the conditions of its occurrence, the disposition of its colors, the poetical meaning which it carries, is not exhausted in any one sunset. The realm of discourse is not the qualitative realm, even though discourse may and ultimately must be about the latter. In so far as qualities are talked about, the immediacy which is theirs as distinctive existences is neglected; and in losing sight of what is so characteristically theirs we are on the road which leads to mathematical physics.

(*b*) The priority which has been given here, so far, to quality is of course an item in human biography. But a more objective sense of priority may be predicated also. If, as will be urged later, the quantitative evaluation or determinateness of quality is significant or definable only in terms of the *operations* of measurement required for achieving that determinateness, and if measurement is a process associated with the activities of men, then quality must be regarded as prior to quantity in both the second and third senses specified above. Nature is measur*able* antecedent to the actual existence of processes of measurement which bring to fruit that possibility, although the meaning of that possibility lies in the eventual performance of appropriate acts upon and within a diversely qualified subject matter. A quantity which is the quantity of nothing in particular, unless it is the quantity of something in general, is meaningless. The evaluation of ten pounds as the weight of one object, or five feet as the length of another, demands well defined processes which carry out those estimations. The occurrence of such processes, however, is subsequent in time to the occurrence of heavy and extended things. The processes are also subsequent logically, for they require intrinsic existential interconnections of heavy things and extended things which condition the performance of the measuring operations. The adequacy of any measurement is defined by a subject matter which controls the relevance of thought and action, a control which is manifested in the existence of immediate qualitative distinctions and orders. For if the discursive thought were not accompanied by the non-discursive, immediate presence of the qualities of subject matter dominating the process of analysis, the relevance and efficacy of cognition for subsequent action would be difficult to explain. It is this twofold priority of subject matter, non-cognitively present, to any formulation of it, that is at the bottom of Aristotle's declaration that although the fact of a man's existence involves the truth of the proposition that he does exist and conversely, yet it is the man's existence that is the ground for the truth of the proposition, and *not* conversely.[2]

Of what is merely immediate, of events as *unique*, nothing can properly be *said*, if one does not wish to sacrifice either the immediate or the intelligibility of one's saying. Therefore of qualities as unique, immediate, and irreducible, discourse can render nothing—it can render nothing because, as part of the flux,

[2]Cf. also Alexander: "A man is a father because he is a male, whose functions have been realized; he does not owe his being a father to the paternal relation, but that relation implies his being a father." *Space, Time, and Deity*, Vol. 1, p. 255.

qualities can only be enjoyed or felt, not talked about. When *listening* to music, discourse about it is superfluous, and discourse cannot *literally* convey the swell and anguish of the sound. For discourse is a recognition of something absent, as Santayana remarked, and is a salutation, not an embrace of existence. Knowledge *is* not its own subject matter. Although existence is both intelligible and sensible, the intelligible is not identically the sensible. The quality of being ill or being green are not conveyed by treatises on pathology or by books on physical optics; at best, these may indicate the routine to be followed in order to recover that which is incommunicable. If, therefore, the world were *wholly* immediate Plato could never refute Protagoras, not because Protagoras would be in the right, but because in such a world refutations could not take place. It is wisdom, consequently, to let quality remain undefined; although what is undefined is not as is sometimes supposed,[3] quality as category, but only the individual events intrinsically qualified. To discuss the locus and function of quality is to define it as a category. The force of all irrationalistic tendencies in philosophy and literature has always resided in the clear perception that discourse, however artful, never exhausts its subject matter, as the map is never the country-side *some* repeatable features of which it symbolizes. Language and thought, themselves possessing those immediate qualities which characterize them as unique existences, must be concerned with subject matter having non-discursive as well as communicable aspects.

It would, however, be mistaking the intent of this essay, if the statement that thought does not exhaust and actually transmit the qualitative wealth of a subject matter should be interpreted, as it sometimes has been, that ideas, being something intellectual, cannot render the characters of existential subject matter. The doctrine that the relations expressed by ideas are also relations between things, has been sometimes classified as a belief in magic; and according to some, "it is difficult if not impossible, to distinguish between applied mathematics and astrology, alchemy and occult arts."[4] As there are dialecticians who affirm that we cannot say that "sugar is sweet" without contradicting ourselves, so there are others who believe that it is impossible that the answer to the question "how much?" can ever be, literally and without a metaphor, "five pounds." Discourse, for such philosophers, can never reach over into the realm that is not discourse and so exhibit some of the interconnections to be found there; only analogically or metaphorically are our ideas ideas of things. The universe, according to them, is divided into a domain of things whose mutual relations are forever out of the realm of discourse, and into a domain of terms in discourse whose mutual relations stand over against and outside of the relations between things.

The immediate purposes of this chapter do not require a detailed examination of the argument by which such a scepticism is reached and defended. In essence, the dialectic consists in mistaking the discriminated products of an analysis for its primary subject matter, and then, by a verbal trick, in raising issues touching the existence of the primary subject matter as in Locke, or

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[3]McTaggart, *Nature of Existence:* "If nothing were red, red would still be a quality." Vol. 1, p. 61. It is not a *red event* of which such statement can be true.

[4]Buchanan, *Poetry and Mathematics,* p. 11. See also pp. 76 ff.

the adequacy of analysis *überhaupt,* as in Bradley. It is sufficient for our purposes, however, to turn the dialectic back upon itself.

A recent form of the sceptical argument consists in maintaining that every proposition contains an analogy in some form, and that, consequently, in all such propositions about existence "is like" ought to be substituted for "is". "Man *is* an animal," must be replaced by "man *is like* an animal;" "the weight of this sugar *is* five pounds," must be replaced by "the relation between this quantity of sugar and another, *is like* the relation between five and one." Science, according to this argument, is only an allegory that asserts without much justification that the relations between parts of reality are *similar* but never identical to the relations between terms of discourse.

Now it is true that if knowledge is taken as a process, it *begins* with a metaphor not recognized as such, although it *moves* toward the elimination of those characters which appear to be merely "like" in some vague way, and *arrives* in the final formulation at the identical relational elements which make the subject matter intelligible. The presence of such identical elements cannot be doubted by the sceptical dialectic itself. For if "is like" must be substituted for "is" in all propositions about existence, how are we to understand the argument itself? Is the argument, too, a metaphor? When we are told that science is an allegory, are we to understand that it is only *like* an allegory, but not quite? Can the argument, on pain of endless regress, ever say more than that "science is—science?" Discourse, too, is subject matter for further discourse. Do we never get the essence of the first discourse in a second one? But above all, if there is *no* sense in which there is an identity between idea and object, how is the peculiar "working property" of some analogies explained, especially their power to predict? Why should some metaphors stretch and break? If existing things *suggest* some intelligible forms to us, as it is admitted they do, is it not perchance because the things themselves possess them? How could a community of interest and action be established, or the communication of ideas be effected, unless ideas were adequate to their objects; unless, that is, "the order and connection of ideas were the same as the order and connection of things?"

Subject matter can never be the goal of inquiry, since every inquiry must begin with it and be docile to its leadings. The qualities which pervade the subject matter are the guide and check for every analysis made. To say this, is to say that the operations of thought go on in a segment of existence distinguishable from other segments by a quality which makes of this segment a determinate event, an individual in some sense. It is also to say that the individuating quality characterizes the relation of elements distinguished within this segment, as just *that* relation of elements which constitutes *this* segment. The test of the adequacy of any empirical analysis consists in a reference to immediate qualities which control and define the distinctions made.[5] Analysis has an inevitable teleological character.

An illustration may perhaps make this clear. Water, considered as a set of qualities eventuating on various occasions, may become a subject matter for inquiry upon the discovery that many of these qualities disappear when, for example, an electric current is passed through them. We are led by various

[5]Dewey, "Qualitative Thought," *Symposium,* Vol. I.

considerations to regard water as a structure of atoms of hydrogen and oxygen combined in definite ratios under definite temperatures and pressures. Now to be scientifically as well as practically valuable, atoms of *H* and *O* must be capable of definition in abstraction, not explicitly involving their combination as water: we must be able to identify *H* and *O* in comparative isolation, or under different temperatures and pressures, or in their reaction with other elements and compounds. But the adequacy of the analysis for the case of water depends upon our ability to identify *H* and *O*, defined in abstraction from their relation in H_2O, with parts of water defined in terms of the qualities which *water* possesses. The qualities which water possesses as an *unanalyzed* whole regulate the choice of the elements and relations in terms of which water is to be reconstituted as an *analyzed* whole. The relation between the analytic parts defined in abstraction, and the whole into which they enter is a teleological one; the relation cannot be said to be a logical one, if Hume's lesson is well learned. For there is no way of showing that structure *logically* implies the qualities of the whole whose structure it is. Although it may be possible to predict, from a knowledge of the relations between the components of *H* and *O*, that under certain conditions *H* and *O* will unite to form a compound, there is nothing to indicate, for example, that the characteristic wetness, translucency, smoothness, temperature, color, smell, of this compound could be *fore*told. Only when that compound has come into existence, will the qualities which *it* possesses be capable of defining the elements of *its* structure. If it be urged in opposition that from a knowledge of the configuration of the wheels, springs, etc., of a clock which one has never seen before, and of the laws of mechanics, one can as a matter of fact predict the behavior of the mechanism, it is clear that this can be done only on the assumption that the mechanism is a *clock*, one of a set of machines identical in their operations. The need for immediately apprehended qualitative homogeneities in terms of which distinctions made in one context can be reapplied in others, is inescapable, even though the *meaning* or *reference* of these qualities is not itself immediate. It is sound metaphysics and logic which picture Wonderland as so puzzling to the casual visitor.

When qualities are not identified exclusively with the material, temporal characters of things, conveyed like color through the senses or imputed by inference through observing sensible qualities, confusion is often introduced. Quality has been taken sometimes to denote both the specific, temporal traits *qua* specific, and those logical and categorial differentia which define properties within a discursive system. In Hegel, for example, beginning with quality as abstract determination, the progressive supplementation and modification of it in the course of controlled inquiry, has been interpreted as the emanation of the concrete from the abstract. But the difference between the differentia of things as undergoing change, and the differentia of things as caught up in the network of inquiry, cuts deep. The latter are logical or essential determinations laboriously selected for their cognitive value; the former constitute the inexhaustible flux from which selection is made. So an *essential* difference between man and fish, classified as animals, is that the former is mammate and lung-breathing, the latter egg-laying and gill-breathing; for this difference enables us to organize and survey a very diversified group of living things. But the countless characters of physical, chemical and vital nature which men and fish

possess as *existing* things, have a fulness, an immediacy, and a physical efficacy, of which the classifications of the zoologist are only a very meager account. The intellectual and moral qualities of men, the qualities of being just, healthy, passionate, irritable, timid; the physical qualities of being soft, hard, hot, dry; of being short, thin, erect, must first be realized in existence, before the relational character of discourse about men can supervene to rob existence of its immediate, compelling flavor. "Existence," in the words of Professor Woodbridge, "cannot be antedated." To deny that there is ineffable quality in the objective realm of nature[6] and to affirm that objectivity only of quantity which is a *relation,* is to deny the relevance of relations to the world which is most obviously the one in which we live.

Consequently, the twofold priority of quality here discussed, recognizes that qualitative events in their immediacy are specific and discrete, and yet possessed of features which make them capable of entering discourse and measurement. Metaphysically, qualities are individuating, giving nature an aspect of discontinuity, and yet in other contexts unifying, disclosing nature to possess homogeneity as well. The quality or qualities which physically set off nickel from iron, is the quality which identifies repeated instances *as* nickel. It is by forgetting or overlooking the stretches of homogeneity which qualities possess, *within* which further distinctions can be made, that the introduction of quantity is for some, like Schlick, a complete "Schoepfung des menschlichen Geistes."[7] Only by regarding space and time as continuous *receptacles* containing discreta, can he explain the measurability of quality. In the flow of qualities, the power of nature is actualized; but the qualitative individuals, by enduring in time, exhibit stretches of qualitative identity which is their *conatus.*[8] Did not the endurance of objects, did not their spatial and temporal distribution involve the existence of qualitative homogeneities, it would be impossible to recognize them as *those* objects; and it would be impossible to carry over to new contexts distinctions and principles adequate in others.

(*c*) If the priority of quality may be affirmed in the first three senses enumerated above, only confusion results if the fourth sense is also affirmed of it. In so far as quality is the specificity of the flux, quality cannot be more general than anything else. The *particular* eventuation of qualities is not the invariable formulation of their appearance. Universality is discovered only when the immediate *qua* immediate is left behind.

The remoteness of physical theory from common-day analyses must be understood as the outcome of a symbolic rendering of relational patterns which find illustrations and specific meaning in immediate subject mater, but which can never be completely identified with any one subject matter. The partial differential equations of the second order which are the general equations of wave motion, represent features in the propagation of disturbances of heat, light, electricity, magnetism, sound, etc. If their meaning in terms of the specific qualities of one context is not their meaning in another context, there is nevertheless a formal pattern discoverable in all contexts so that the several

[6]Cf. Von Hartmann, *Kategorienlehre,* p. 42.

[7]*Vierteljahrschrift f. wissen. Philos.* Vol. 34, p. 141.

[8]Qualitative continuity remains the basis for the procedure of applying the theoretical structure of science to existential subject matter. Cf. Russell, *Analysis of Matter,* p. 82.

contexts may be regarded as exemplifications of one relational structure. It is by measurement that science achieves the translation of what is immediate into what is not so. The relational nexus in which qualities are found is expressed in such a manner, that the distinctive features of any one context drop out. Consequently, the magnitude or measure of a property is never the property itself, but a logical function of the relations in which that property is exhibited. That is why the number which designates the length of a rod is not the spatial *quale* of the rod, but may be a function of its velocity or the time of its measurement.

More detailed illustration of the process whereby a general formulation of qualitative events is achieved will be supplied later. But two difficulties that have been commonly encountered because of such formulations will be briefly mentioned at this point.

(1) It is by demanding substantiality and constancy in the wrong place that many of the puzzles of the philosophical interpretation of science have arisen. The task of science as a search for an inner, non-relational structure in things, to be apprehended immediately, misrepresents its function and method. Moreover, if thinghood or substantiality is taken as residing in something which is immediate and particular, causality is often denied outright (Wittgenstein), since it is as a relation of events that causality is a scientific category; or it is regarded as an efficient agent residing in some qualities like extension and periodicity, but not in others like color or sound. Qualities like the latter are interpreted relationally, as dependent upon non-relational characters, and become dishonored metaphysically; while those like the former are taken as intrinsic, and raised to metaphysical glory.

The tendency to look for substance in the immediate and particular, is at the bottom of all those views which find in atoms an individuality "higher" than that of everyday objects. "When it comes to the last analysis, the only ultimate and absolute true thing would be an atom that is absolutely changeless and eternal . . . Substance is that ultimate, eternal, unchanging element in our world toward which science in her atomic theory, keeps pushing."[9] To pursue this road in all rigor, is to conclude that the work-a-day world is not a legitimate foreground from which to survey nature. It is moreover, to find substance where science never finds it; for it is not in the possession or location of immediate quality or unchanging *thing* that science stops; it is rather in the discovery of uniform *relations* between what is immediate and particular. Substance may be defined as that which remains unchanged throughout time and space. But there is certainly nothing in modern science to warrant the belief that it is a particular quality or group of qualities which possesses this character. Substantiality belongs to the *order* of succession between qualities.

(2) If, then, it is relational structure with which science, like all discourse, occupies itself, science never gives any final accounting for the number of qualities the world contains. Yet one of the most curious arguments often used to reduce the wealth of qualities to combinations of a selected few, is found in many textbooks of physics. So we read: "We can obtain information about the mechanism of attraction if we establish the law of force between two masses to be $G\frac{m_1m_2}{d^2}$ and then from the dimensional equation find the nature of the

[9]Marvin, *Introd. to Systematic Philos.*, pp. 215-216.

quantity G. Thus, $[MLT^{-2}]=[GM^2L^{-2}]$; $\therefore$ $[G]=[M^{-1}L^3T^{-2}]$. Owing to our ignorance of the mechanism of gravitation, the dimensions of G are unintelligible, as for example, we can attach no meaning to M^{-1}, the inverse of a mass; but should we obtain knowledge which enables us to express G in terms of some other effects, this unintelligibility will doubtless disappear."[10] How valueless such an argument is for supposing that some qualities are eliminated from nature by reducing them to "mere combinations" of others, is very patent when it is remembered that what the equations of physics assert is a uniform association between qualities. Is it intelligible to say that $[V]=[LT^{-1}]$, if this is to mean that velocity *is*, literally, length divided by time? Dimensional equations supply some information about the way in which magnitudes enter our equations, but they do not even reflect, no less exhaust, the qualitative nature of the things whose magnitudes they are.[11]

In this section, some of the senses in which quality may be regarded as primary and one sense in which it may not be so regarded, have been examined. We turn now to a defense of the ontological character of quality.

II

Philosophers have not always been willing, indiscriminately, to grant definite ontological status to all qualities even though that refusal, when its consequences are drawn, often makes nonsense of customary practise and morals.[12] It is perhaps significant that few of the objections to the full reality of quality have been made on moral grounds. The objections that have been made, however, have gone all the way from regarding qualities as subjective or mind-dependent, to regarding them as self-contradictory and therefore unreal. Some of these arguments will be briefly reviewed.[13]

(*a*) One of the oldest and most frequent attacks on the objectivity of quality has come from the discovery of the dependence of qualities upon the disposition of the percipient apprehending them. For a time, some qualities, the so-called primary ones, were believed to be free from such dependence; but since Berkeley, at any rate, it has become evident that if any qualities are to be discredited because they are conditioned by the perceiver, then no distinction in favor of the primary qualities can be made. If variability constitutes a criterion of subjectivity, all qualities must be regarded as equally subjective. And in particular, if secondary qualities are discounted because supposedly mind-dependent, so must the primary.

There can be little question, however, of the fact that the alleged dependence of qualities upon the mind is really a case of dependence upon the organic *body*. The water which is warm to one hand and cold to another has these qualities in relation to *hands* physically different, not in relation to a *mind;* and such

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[10]Starling, *Electricity and Magnetism,* p. 385.

[11]On the value of the arguments from dimensions, see Campbell, *Measurement and Calculation,* ch. 13, and *Handbuch der Physik,* (Geiger und Scheel) Band II, p. 11 ff.

[12]Rampion, one of Aldous Huxley's characters in *Point Counter Point* declaims with a fine frenzy: ". . secondary qualities have no real existence. The man who takes that seriously denies himself, destroys the whole fabric of his life as a human being. For human beings happen to be so arranged that secondary qualities are for them, the only real ones. Deny them and you commit suicide."

[13]Cf. *Frischeisen-Koehler, Viert.f.wiss.* Phil. Vol. 30, p. 271 ff. and *Annalen der Naturphilosophie* Vol. 6, p. 295 ff.

relativity of quality can be duplicated with *inorganic* physical systems as well. Mind or consciousness is never needed in physics as an explanatory category for the occurrence of secondary qualities. As far as the status of quality is concerned, all that the above mentioned discovery proves is that the quality which supervenes upon the interaction of some objects and an organism is not necessarily the quality which exists without that interaction. It does not prove that the objects are characterized by no quality whatsoever. Consequently, if the familiar qualities of our world are partly dependent for their existence upon physiological conditions, nevertheless those qualities are not something mental, but are qualities of a complex physical process in which the body, not some mysterious "consciousness," is an important ingredient. Both the external object and the organic body are necessary, as far as the argument shows, to achieve the quality. There is as little reason to say that the organism produces the quality as there would be to maintain that it is the lemon alone, or the sugar alone, or the water alone, or the tea leaves alone which produces the unique flavor of tea.[14]

In general, therefore, the discovery that the organic body enters actively into sense perception tends to show that objects not *actually* perceived may possess qualities different from those they do when inter-acting with an organism, and that the former qualities stand to those supervening in that interaction as virtual or potential qualities to the qualities actualized in sense perception. Subjectivity in this connection must be interpreted not as mind-dependence, but as a consequence of the relativity of physical processes. To say that an *object* is white, for example, may mean only that when normal observers are in a certain position with respect to the object, the quality which characterizes a *particular mode of interaction* between observers, objects and the physical medium, is white.

It follows, however, that if there is no way of circumventing the influence of the organism upon sensory qualities, then a number of problems sometimes raised about quality become meaningless. If certain physical systems are *always* involved in the actualization of a specific quality, and if it is only this *actualized* quality which can be noted, then there is no sense in asking for the qualities objects possess prior to every instance of their actualization. So if a self-luminous body *A* must be present directly or by reflection, if another body is to have color, it is worse than useless to demand the color which the second body possesses in the absence of *all* luminous bodies; the only legitimate demand would be for the color of the second body in the presence of luminous bodies *other* than *A*.

Similarly, if the sense organs are an indispensable condition for the eventuation of some qualities, it is self-contradictory to ask for the differentia of those qualities in the absence of these organs. If to be red when not perceived *means* to be red when perceived, to ask whether objects can be *verified* to be red if *never* perceived is meaningless: it can never be proved that objects are red unless one is permitted to use one's eyes. This is the body-centric predicament, and whether organic bodies are a necessary condition for the existence of qualities or not, qualities can be apprehended only with bodies. The question as to whether because one must use eyes to see red objects, objects can be said to

[14]See Woodbridge, "Deception of the Senses," *Journal of Philosophy*, Vol. X.

be red when not undergoing interaction with eyes, is in part a verbal one: if the functioning of the *eyes* is an indispensable condition for the existence of color, the answer obviously is no; but if the eyes are only a sample from a more inclusive group of physical systems which must be undergoing interaction with objects to produce color, the answer is yes, provided some other sample from that group is in activity with the objects. If to be an explosive, dynamite must be explod*ing,* then when it is not doing this dynamite may be said to possess a certain capacity which becomes actualized only in the presence of some other physical conditions, e.g. pressure; it is not, by definition, an explosive when the pressure is not applied, except in a proleptic sense. Hence the character of being an explosive is defined only within a context including the applied pressure as well as the stick of dynamite. The quality of explosiveness is subjective or internal to this situation. All our knowledge then points to the conclusion that *all* qualities are subjective, in this sense, within certain contexts, and that they are objective or relatively independent of other contexts.

Furthermore, since sensory qualities can be noted only by using our senses; and since in the nature of the case we can never discover whether the senses are an indispensable condition or only a sample from a more inclusive set of indispensable conditions for the production of the qualities called sensory; inquiries as to whether the universe is colored, sounding and odorous when not seen, heard, or smelled, must be judged as verbal. Whether we say that the world is so characterized or not, the consequences of our regarding it in either way can lead to no *sensible* differences. The pragmatic rule can be invoked here with some profit. Nevertheless, the belief that the world is qualitatively so diversified is made plausible at least both by the use of the continuity principle that the human scene is a segment continuous with the rest of nature; and by the existence of different sense organs, since the diversity of these organs is consonant with the diversity of qualities. The attempt to cut under this plausibility by Müller's theory of specific nerve energies has been remarkably unsuccessful. No investigation has been able to discover either the specific nerve impulses, or the specific neural chemistry either in nerve fibres or lobes of brain which the theory demands; on the other hand, the information gathered seems to substantiate the belief in the continuity between periodicities in the physical stimulus and the periodicities in the nerve impulse.[15] There is little reason to suppose, therefore, that there are fewer qualities in nature than we apprehend; there are good reasons to think that there are many more.

(*b*) A second line of argument advanced to exclude almost all qualities from the domain of nature, is that the more advanced sciences, like mathematical physics, have no need to recognize them, and that therefore, by Occam's razor, all qualities except a specified few can be shaved off. The more highly developed sciences, it is urged, take note of only certain extensive properties of the material world, in terms of which all other qualities are explained: these other qualities are either "reduced" to complexes of the primary ones, or the primary qualities are taken as causative powers in the production of all others.

But from the fact, that in the explicit formulations of physics most of the

[15]Holt, in *New Realism,* pp. 314 ff.; Frischeisen-Koehler, *Ann.d. Naturphil.,* loc. cit. p. 339 ff.

familiar qualities do not appear, it cannot be concluded that the existence of these qualities is unessential for those formulations.

(1) The so-called primary qualities with which physics does occupy itself, enter into the mathematical equations not as qualities proper, but as relations. These relations require, as will be seen below, qualities to relate. But if only the *explicit* contents of the *equations* are to be regarded as necessary for scientific procedure and therefore as metaphysically real, no qualities whatsoever can be so regarded. This argument, when pressed, leads to the affirmation of the exclusive reality of relations.

(2) The formulations of physics are very largely in terms of spatial and temporal characters, and the qualities associated with these are obtained primarily from optical and tactual sources. The qualities associated with other sense organs do not appear, for example, in analytical mechanics. Consequently, qualities associated within the domain of characters from which the elements of analysis have been taken, are *implicit* in the formulation. "The swinging tongue of the bell must be associated with *some* color, as long as it is thought of in visual terms, even though smell and sound, from this point of view, can be thought away."

(3) There is no good reason given for classifying as non-real, those qualities which have not as conspicuous roles as others in physical inquiry, even if it be admitted that the latter qualities are physically more efficient than the former. Why should periodicity, even if regarded as a causal agent, be metaphysically superior to color? As a matter of fact, however, the equations of physics do not indicate which are the efficient qualities of matter. They assert relations of coexistence and succession between qualities. If some qualities *seem* more predominant than others (but see the comment under 1), this is partly because they are more pervasive and easier to get at than others, and partly because, as will be seen, they are *logically* prior in the process of measurement, not prior in existence or efficient power.

(4) In a deductive system, the elements in terms of which others are defined are chosen primarily for the ease with which they forward the deductive process. They possess no intrinsic superiority over the defined elements, and the respective roles of *definiens* and *definiendum* can often be reversed. Lines can be defined in terms of relations between points, are no less real than points, and can never be "reduced" to points alone; points may similarly be regarded as functions of lines. In the same way, the discriminated elements in terms of which the variations of a qualitative whole can be best expressed and controlled, cannot be used to deny a status as objective as they themselves possess, to the wholes of which they are parts. The spatial, temporal, and inertial characters in terms of which the flight and destructive agency of a projectile are calculated are not therefore evidence for the lesser reality of the flight and of the havoc wrought.

(5) In the *application* of the concepts of theories to existing subject matter, no one will deny that many of those secondary qualities of which the theory makes no *explicit* note, *must* be used in order to identify the objects studied and to give a physical meaning to the theoretical structure. One must, therefore, accept these secondary qualities as the primary subject matter of physics, about whose existence and objective reality no final doubt can be raised without stulti-

fying all science. For the concepts of physics are fashioned to coordinate those qualities; to deny them an ultimate status in reality, even though that status is different in kind from the relational structures, is to make of physics an uncontrolled fantasy, and to deprive it of all capacity to survey any portion of the real.

(*c*) A third kind of argument against quality takes a dialectical form, and concludes that reality cannot be thought without contradiction to possess quality.

(1) Qualities, one variation of the dialectic runs, always come to us in relations, and without the relations in which they stand to one another they cannot be thought intelligibly. But as related, qualities are constituted by their relations, and the internal unity of qualities is dissolved into an endless regress of relations. Relations without qualities to relate, however, are as unintelligible as qualities without relations; although relations cannot relate qualities without interposing an endless regress of relations, relating relations to qualities. "Relation presupposes quality, and quality relation. Each can be something neither together with, nor apart from, the other."[16]

Now indeed, if thinking is relational, if it is not the grasping of some inner essence but the discovery and pursuit of connections between things, then to think of quality is to think of it in some relation—as distinct from or similar to other qualities, or as conditioned in its occurrence by many other factors. But from this observation it does not follow that qualities are dissolved into relations, even though they always exist in some relation to others. The whole force of Bradley's argument consists in confusing the intellectual or inferential activity, with the non-reflective activity of enjoyment or possession. To *say* that something is quality, is already to have left the brute immediacy of existence and to have reworked it into the intellectual distinctions of discourse. To be sick, to be red, to be odorous, is compatible with the presence of certain uniformities between these characters and others with which they coexist; but these uniformities are not the characters any more than the toxin produced by microorganisms in sickness, or a vibration is the color red. Characters which in *reflection* are distinguished as qualities and therefore involve relations in any intelligible *statement* of them, can be possessed non-reflectively and non-relationally: the flushed skin may *signify* fever, but as an existential quale it is a character of the organism possessed by it immediately and "enjoyed" non-relationally. The irreducible "thisness" or particularity of events and characters need not and cannot be ruled out of existence, just because discourse does not and cannot convey the incommunicable. "If anyone asks me, what this solidity is," wrote Locke at the *end* of the chapter discussing this quality, "I send him to his senses to inform him."

Furthermore, since Bradley himself declares that the problem of how a relation can stand to its qualities is insoluble, one may suspect with some right that the insoluble problem has been manufactured by reifying the relation, which then no longer relates. The whiteness of sugar is related to its sweetness. How related? In this manner at least, that that which appears white to the eye and hard to the touch, will be sensed as sweet when taken into the mouth. Green is different from red, and therefore related to it. Again how related? In

[16]Bradley, *Appearance* and *Reality*, pp. 26 ff.

this respect at least, that in any grouping of objects with respect to color, there is a class of red objects and a class of green objects which are not the same class. Any other sense of "how" is difficult to make out, unless the relation of "Being-on-a-man's-head" which holds between hat and head, is taken to be as space filling as the hat; unless, that is, the relation is falsely taken to be another thing.

(2) A second variation of the dialectic against quality has used the distinction between the thing and its qualities to force a recantation of the belief in the ontological status of the latter. When an unqualified substratum is construed as the thing owning the qualities which inhere in it, the unknowable substance has often been taken as the real, and its phenomenal qualities as appearance; or the unity of the thing has been challenged by pointing to alleged violations of the principle of excluded middle, if the thing is taken as having many qualities. How the same thing can be both elliptical and round, cold in varying degrees, or sounding in several pitches, are questions often believed to be fatal to the distinction.[17]

Now the conception of a "thing" is a practical one, testifying to the successful nature of certain habits which isolate *some* qualities as the more important ones for the purposes of controlling the eventuation of other qualities uniformly associated with the first set. It is because of this tendency to take the more easily manipulable traits as definitions of a thing, that spatial definiteness has received so large a share in the analysis of things. The difficulties that arise from locating things in clearly demarcated spaces and times, have received much attention, while more recently the practice has been baptised as the fallacy of misplaced concreteness.

Since a red apple, for example, possesses this color in hues and shades which are functions of the points of observation and of the physical medium, as well as of the surface of the apple, a relational way of thought must attribute thinghood not primarily to the surface and volume of the apple; a relational way of thought must identify the apple with all the colors, odors, tastes, etc., appearing in all those relations which set off these qualities as constituting one thing within some specified context. Qualities like heat, odor, magnetism, which are often predicated of a "thing," show how compatible ambiguity of spatial location may be with its integrity as an individual. Some of these qualities are taken as conventional signs of others, and are often used to define the thing; recognition of this makes it unnecessary to believe in a non-experienceable substance, or in a mysterious relation of inherence.

(3) The variation of qualities with the circumstances in which they are found, has been the occasion for some attempts to eliminate specificity altogether by reducing qualities to complexes of relations. The observed height of a building has a unique spatial quale which varies with the conditions of observation; but when the height is taken as a term within a discursive system, it often becomes transformed into relations of comparison, or into a series of measurements. What more natural, then, than to regard individuals as only the nodal points in the convergence of relations? The brilliant success of the sciences

[17]See e.g. Schlick, *Allgemeine Erkenntnislehre*, 2te Auf. p. 243. For the analysis of the next few paragraphs, see M. R. Cohen, *Jr. of Phil.* Vol. XI, p. 617 ff.; P. Nunn, *Aristotelian Proceed.* Vol. X; and Whitehead, *Science and the Modern World*, p. 72. ff.

in discovering invariant features in successions and coexistences, has invited many to think of existence as a summation of universals, and of individuality as a derivative character.

To pure intelligences, intent on pursuing the dialectic way, such a universe is doubtless the most delectable one. To them, since they would never have occasion to leave the realm of essence discourse would be the only activity, and essences its only objects. Discourse alone could never exhibit a non-discursive realm. But if dialectic alone can never convince men, their animal bodies and habits invariably do, that universals and relations do not constitute an adequate inventory of the constituents of the universe they inhabit.

Yet even the doctrine which attempts the reduction of qualities to relations, must recognize that there are other entities in the universe than relations, since the center of convergence of relations is not a relation and may conceivably have a quale, which although conditioned by the relations it has to other such points of intersection, is nevertheless not a relation. To admit so much is in essence to admit the irreducible nature of quality, and its claim to a metaphysical rank coordinate with relations. There need be no unrelated qualities, even if there is something in a quality like red which is not a relation.

If, therefore, we regard qualities as the eventuation of potencies and directions which other groups of qualities manifest, qualities possess an incommunicable immediacy and compulsion which the terms of discourse, whose extension they are, do not share. It is at least a workable hypothesis, that in all intellection two irreducible factors are present: logical distinctions, the heritage of previous inquiries, in terms of which the ongoing investigation is moulded and directed; and ineffable qualities, constituting one aspect of the subject matter explored. However inextricably interwoven, both factors seem necessary if knowledge is ever to be knowledge of existence, or if knowledge is ever to be something more than existence.

From these general considerations upon the function of quality, and upon the arguments which entitle qualities to a metaphysical ranking, we turn to the conditions and processes of scientific measurement.

CHAPTER II

MEASUREMENT

I

The occasion and conditions for measurement. Measurement has been defined as the correlation with numbers of entities which are not numbers.[1] As practised in the developed sciences this is a sufficiently comprehensive, though cryptic, statement of the object of measurement. But in a larger sense, in a sense to include most of those acts of identification, delimitation, comparison, present in every day thought and practise, numerical measurement is only infrequently used. "*This* is the missing book," or "He had a *good* sleep," or "The cake is *too* sweet," are judgments making no explicit reference to number. From this larger point of view, measurement can be regarded as the delimitation and fixation of our ideas of things, so that the determination of what it is to be a man or to be a circle is a case of measurement. The problems of measurement merge, at one end, with the problems of predication.

There are indeed vast domains of reflection and practise where numbers have taken little hold. Prior to Descartes, geometry was not established on a thoroughgoing numerical basis, and many branches of mathematics, like symbolic logic or projective geometry, may be pursued without introducing numbers. In arts like cookery, measurement is not primarily numerical, and the operations used are very often controlled by disciplined judgments on the qualitative alterations of the subject matter.

But the difficulty and uncertainty often experienced in obtaining desired consequences when vaguely defined ideas and crude methods of applying them are used, soon lead wherever possible to the introduction of more or less refined mathematical processes. The immediate, direct evaluation of subject matter secures too little uniformity to be of much value; and direct judgments, e.g. of lengths by the eye, are consequently replaced by more complicated and indirect operations, such as the transporting of unit lengths along the lines to be compared. It is, indeed, because less error, that is, greater uniformity, is obtained in judgments of spatial coincidences than, e.g. in judgments of differences of length or color, that spatial congruence plays so large a role in laboratory practise. The *raison d'être* of numbers in measurement, is the elimination of ambiguity in classification, and the achievement of uniformity in practise.

It is generally only after numerical measurements have been established and standardized, that references to the "real" properties of things begin to appear: those properties, that is, which appear in circumstances allowing for most facility in their measurement. The "real" shape of the penny is round, because from the point of view from which the penny is round, measurements and correlations of other shapes can be carried on most easily. Nevertheless, it must not be overlooked that a numerical evaluation of things, is only one way of making evaluations of certain selected characters, although it is so far the best. It is preeminently the best, because in addition to the obvious advantage they have as a uni-

[1]Spaier, *La Pensée et la Qualité*, p. 34.

versally recognized language, numbers make possible a refinement of analysis without loss of clarity; and their emotionally neutral character permits a symbolic rendering of invariant relations in a manifold of changing qualities. Mathematics expresses the recognition of a necessity which is not human.

From this last point of view, therefore, the search for a unified body of principles in terms of which the abrupt, the transitory, the unexpected, are exhibited and in a measure controlled, is the most conscious guiding principle in the application of numerical science. If the search for mathematical equations is the aim of physics, other activities such as experiment, classification, or measurement are subservient to this aim, and are to be understood only in relation to it. Consequently, if we inquire why we measure in physics, the answer will be that if we do measure, and measure in certain ways, then it will be possible to establish the equations and theories which are the goal of inquiry.

It is relevant, therefore, to demand the logical foundations of the mathematical operations which physics constantly uses. For if mathematics is applicable to the natural world, the formal properties of the symbolic operations of mathematics must also be predicable of many segments of that world. And if we can discover what these formal properties are, since mathematics *is* relevant to the exploration of nature, a physical interpretation *must* be found for them. That physical interpretation will constitute, whenever it can be found, the conditions for the measurement of that subject matter. Consequently, if we ask why in measurement we attend to certain characters of objects to the exclusion of others, the answer will be that the selected characters are precisely those with which applied mathematics can cope. It is only by a reference to the function which the numerical measure or magnitude of things have in equations, that one can remove the apparent arbitrariness in selecting one rather than another set of conditions as fundamental for measurement.

In recent years the formal conditions of their science have been much discussed by mathematicians. The properties which magnitudes must have in order to be capable of the kind of elaboration which the mathematics of physics requires, have been variously formulated as axioms of quantity.[2] The following set, with some modifications, is taken from Hoelder.[3]

(1) Either $a>b$, or $a<b$, or $a=b$.
(2) If $a>b$, and $b>c$, then $a>c$.
(3) For every a there is an a', such that $a=a'$.
(4) If $a>b$, and $b=b'$, then $a>b'$.
(5) If $a=b$, then $b=a$.
(6) For every a there is a b such that $a>b$ (within limits).
(7) For every a and b there is a c such that $c=a+b$.
(8) $a+b>a'$.
(9) $a+b=a'+b'$.
(10) $a+b=b+a$.

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[2]It need be mentioned only in passing that mathematics cannot be regarded as exclusively the science of quantity. Its essence is the study of types of order, of which the quantitative one is a single instance.

[3]"Die Axiome der Quantitaet," *Ber. d. Saechs. Gesellsch. d. Wiss., math.-phys. Klasse,* 1901.

(11) $(a+b)+c=a+(b+c)$.

(12) If $a<b$, there is a number n such that $na>b$ (also within limits)[4].

How much more perspicacious such an axiomatic or functional analysis of magnitudes can be than definitions in terms of private, intrinsic properties (e.g. "quantity is the relation between the existence and the non-existence of a certain kind of being");[5] how much more successfully it satisfies the demand for a formulation which, true for *every* instance of quantity, should not be the exhaustive statement of *any one* instance, will be evident from the sequel. Even Meinong's definition of magnitude as "whatever is capable of being limited toward zero" or as "that which is capable of having interpolations between itself and its contradictory," has little clarity to recommend it.[6] Later on a distinction will be made between magnitudes which satisfy all twelve axioms and those which satisfy only the first six. A magnitude in the most complete sense, however, is whatever is capable of verifying the whole set.

II

Order and equality. The illustration of these relations by indicating some of the empirical procedure involved in measurement, is the next portion of our task. At the very outset it must be pointed out, however, that an adequate interpretation of the axioms must inevitably lead out of the laboratory where measurements are usually made, and lead on to a consideration of the manufacture of the laboratory *instruments*. When a laboratory experiment is studied behavioristically, the measurements performed consist in the observation of the movement of a pointer on a scale, or of the superposition of lengths.[7] How natural, therefore, to suppose that measurement consists only in the observation of space-time coincidences! It is important to remember, however, that the experimenter, working with marked or calibrated instruments, assumes that the calibrations indicate various qualitative continuities not *explicitly* present. The process of measurement has not been fully exhibited until all those operations of calibration have been noted. When a weight is attached to a spring balance, and the position of a marker on the scale read, only a very small fraction of the process actually necessary to estimate the weight as five pounds has been observed; the operations entering into the construction and *correlation* of scale and spring must be included. It is of the essence of an experiment that it be repeatable. Therefore it is not the particular instrument used any more than it is the unique experiment which has such an overwhelming importance in science; it is rather the repeatable process capable of producing the markings on the instrument which is. Every marked instrument implies the construction and existence of some standard series of magnitudes, correlation with which constitutes the calibration. A whole-hearted recognition of this reference of instruments to something beyond themselves, is a recognition that other characters of

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[4]On the importance of the Archimedean axiom, see Hilbert, "Axiomatisches Denken," *Math. Annalen,* Vol. 78, p. 408. The fact that one can estimate stellar distances by adjoining earthly ones, is not a logical consequence of theorems on congruence, but is an independent experimental conclusion.

[5]So Warrain, a follower of Wronski, in *Quantite, Infini, Continu,* p. 9.

[6]"Uber d. Bedeutung d. Weber'schen Ges." in *Gessam. Abhandl.* Vol. 2, p. 219.

[7]Cf. Duhem, *La Theorie Physique,* p. 219.

existence besides the spatial are capable of, and are involved in, the process of measurement.[8]

The relation $>$, or the converse $<$, is a transitive assymetrical relation. It finds its exemplification in some discovered qualitative domain which is sufficiently homogeneous to allow identification as a well defined range of a single quality.[9] Within this domain the character studied must be capable of such a serial gradation that a transitive assymetrical relation can be discovered to hold between the discriminated elements. So we find the character of density which liquids manifest in relation to one another to be such a relation. This character may be defined, with somewhat more care than is shown here, as the capacity of a liquid to float upon other liquids. Liquid a will be said to be more dense than b if b can float on a but a cannot float on b. And it can and *must* be shown *experimentally*, that, for a set of liquids distinguishable from each other by all sorts of physical and chemical properties, the relation $>$ (more dense) is a transitive assymetrical relation: if liquid a is more dense than b (i.e. b floats on a) and if b is more dense than c, then a is more dense than c (i.e. c floats on a.)

The relation of equality ($=$) can be defined in terms of $>$. We say that $a=b$, if a is not $> b$, and a is not $< b$, and if $a \gtrless c$, then also $b \gtrless c$. Equality of density may therefore, be defined thus: c has a density equal to that of d if c does not float on d and d does not float on c; and if c floats on e so does d.[10]

The experimental establishment of series of this kind involving the first six axioms is the first step in the introduction of number. The function of the experiments is the careful exhibition of physical relations symbolized by these axioms; without such an experimental exploration of the subject matter, it may turn out that the relation between the objects under survey will not generate the transitive assymetric sequence. If, to take an absurd example, we compare the lengths of elastic rubber bands by superposition, without specifying further the manner in which this is to be done, and if we define x as longer than y when x extends beyond y, it may well be that axiom two will not be satisfied.

Numbers may of course be introduced merely for purposes of identification, and many numerical designations have no more arithmetical significance than do the names of individuals. Thus, the policemen of a large city are often known by their number; but the policeman with number 500 is not thereby known to be stronger, or more efficient, or more handsome, or wealthier, or older than the one numbered 475.

This kind of out-and-out arbitrariness can be considerably reduced if a qualitative series is first established. The numbers assigned in the operations

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[8]Dingler, in *Das Experiment*, p. 51 ff., has an important discussion in this connection, whatever one may think of his characterization of theoretical physics as without a physical meaning.

[9]Runge, "Maass u. Messen" in *Encyk. d. Math. Wiss.*, Bd. V, p. 4.

[10]This definition of equality in terms of the relation generating the series is due to Campbell, *Measurement and Calculation*, p. 5. The many debts of this paper to Campbell are evident to all his readers, and cannot be made explicit even by repeated citations. The discussion in Duhem, *Theorie Physique*, p. 159 ff., takes similar lines, while Helmholtz's important essay on *Zählen u. Mässen*, contains the germ of almost all subsequent works on the subject. The essay is reprinted in his *Schriften zur Erkenntnistheorie*, edited by Hertz and Schlick. "Equality between the comparable properties of two object is an exceptional occurrence, and can be recognized in empirical observations only by this, that the two equal objects make possible the noting under proper conditions of a special effect, not usually present in the interaction of other pairs of similar objects"; p. 85. See also Cournot, *Essai*, p. 286 ff.

of science are more than a conventional tag; it is desirable that an object numbered 50 occupy a position higher or lower in a qualitative series as determined above, than one marked 40.[11] So, for example, the liquids gasoline, alcohol, water, glycerine, hydrochloric acid, carbon bisulphide, and mercury, are arranged in the order of increasing density as defined above; we can establish a one-to-one correspondence between these liquids and a series of real numbers such that the order of numbers will be symbolic of the order of densities. We may assign any one of the following sets of numbers to the series of liquids: (a) 1,2,3,4,5,6,7, or (b) 100,90,88,85,80,60,10, or (c) 22.5, 20, 19.6, 19.3, 11.2, 10.5, 8.9, or (d) .75, .79, 1, 1.26, 1.27, 1.29, 13.6. If, however, we specify further that the order of increasing numerical magnitude must correspond with the order of increasing density, the sets of numbers (b) and (c) are no longer available.

Nevertheless, there is still very much that is arbitrary and often misleading in assigning the numbers, say, 1 to 7 to the above sets of liquids. In choosing the set (a) instead of (d) or other sets having the same order, we exhibit the arbitrariness. The choice becomes positively misleading, if, without further definition and experimental confirmation, we suppose that because alcohol in this scheme has a density of two and gasoline that of one, there is involved a physical meaning (a physical operation defined in terms of density) in talking of alcohol as *twice* as dense as gasoline, or of *adding* densities of gasoline to obtain the density of alcohol. The operational interpretation of the axioms assigns, so far, physical meaning only to the *order* in which numbers are employed; there is no such meaning as yet for numerical *differences*.

It is just such misunderstandings, however, that are at the basis of many of the confusions in psychological and social measurements. Bogoslovsky's attempt to measure the proportion of "mental activity elements" in various tasks, is one instance of such confusions.[12] He presented to several people twenty descriptions of different situations involving mental and physical activity, and obtained their estimate of the proportion of mental activity "elements" to physical activity "elements" in each. The judges were asked first to arrange the descriptions in order of increasing mental activity, (an attempt to generate a qualitative series) and then to express the numerical percentage of mental activity "elements" in each. But "percentage of mental activity elements" has a well defined meaning only on the assumption that the situation can be regarded as the *summed* total of elements; and that assumption must be justified both by an adequate criterion for an "element," and by the exhibition of a process of *addition* for them. Without that assumption and its justification, the results obtained indicate nothing. How little the nature of measurement is understood by the author is clear from the defense he makes for his procedure: "All our scales are arbitrary. There are no special intrinsic reasons for dividing an hour into sixty minutes." An hour may, of course, be divided into any number of intervals. But it is the possibility of definite operations defining more or less time and the addition of intervals, that makes the division of an hour into sixty parts valuable, and the non-existence of such operations which makes valueless the computation of mental activity percentages.

When, therefore, numbers are correlated with some qualitative spectrum, it

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[11] Cf. Runge, *op. cit.* p. 5 ff.

[12] *Technique of Controversy*, pp. 162-74.

is not obvious and not always true, that the differences between the numbers represent differences between qualities definable otherwise than ordinarily. Since none of the operations exhibited so far in connection with density have defined anything besides "geater than" and "equality," it is meaningless to talk, at this stage, of addition.

III

Addition. "At this stage" must be amended in the case of some properties to read "ever." For it is a well established character of existence that, although many qualities can be serially ordered and so numbered in accordance with an arbitrary plan, a few of these qualities (and a limited few they are) possess also a capacity for "addition" which the rest do not. This cleavage between additive or extensive qualities, and non-additive or intensive qualities, is of fundamental importance in the philosophy of physical measurement.

Consequently, new operations and experiments must be introduced to define addition. And for density, defined as above, such operations are not obtainable, since there is no clear sense in which two liquids equally dense could be added to produce a liquid twice as dense, and so obtain a "sum" which would possess the formal properties listed in the last six axioms. Other physical properties must be used to exhibit these new operations.[13]

"Illumination" is a very important photometric property which is capable of addition when defined as follows. It will be assumed, in the first place, that the immediate but disciplined judgments, with respect to perceived inequality or equality of the brightness of certain surfaces, can be obtained with sufficient uniformity. Brightness thus forms a qualitative domain within which more or less bright can be distinguished; equality of brightness may therefore be defined in conformity to the formal characters already specified. The surface used will have the same shape, same color, and the same reflection and diffusion coefficients. Surfaces will be said to form a "pair" if, when their positions are interchanged and everything else remains the same, no disturbance can be noted in the equality of the brightness of the surfaces.

A pair of surfaces will be said to have equal *illumination,* when they are judged to be equally bright under specified circumstances: e.g. the lines which join the two surfaces to the eye must make equal angles with the perpendiculars to the surfaces. Many more specified conditions must be introduced in practise. When some body can be found such that, by altering its physical state or of the medium between it and the surfaces, the brightness of the surfaces is changed, the body is defined as the *source* of the illumination. It can now be shown experimentally that if a pair of surfaces form an equally illuminated pair under a certain source, a second pair will also be equally illuminated if substituted for the first pair, even though a member of the first set does not form a "pair" with a member of the second set. This is the reason why illumi-

[13]For the following illustration, see Campbell and Dudding, "Measurement of Light," *Philos. Magazine,* 6 Ser. Vol. 44. The discussion in Lambert's *Photometrie* (translated in Ostwald's Klassiker) is the foundation for the whole science. He shows, incidentally, the self-corrective nature of scientific procedure. "If one wanted to investigate the validity of eye judgments, one must remember these psychological illusions in order to take cognizance of the remaining principles of photometry. But it is these principles that are presupposed in any investigation of errors of eye judgments. Hence I do not see how a logical circle can be avoided if a rigorously proved photometry is desired. But if this rigor is abated a little, one can obtain the propositions of photometry with some degree of certainty." Vol. 1. p. 7.

nation comes to be regarded as a quality not of the pair of surfaces but of the conditions under which they are placed; these conditions include, as a minimum, the nature of the source and the position of the surfaces with respect to it.

Addition of illumination can now be defined. The illumination of a surface S from sources A *and* B is defined to be equal to the *sum* of the illumination of S from A alone and the illumination from B alone, if A and B remain in the same physical state and relative position to S. If we use sources of the same color, (strictly, only monochromatic light can be used) it can be shown that axioms 8, 9, and 11, receive an experimental confirmation; but for heterochromatic light, addition in this complete sense no longer exists.[14]

With physical addition defined, numbers can now be introduced, so that all arbitrariness in assigning them, except in the choice of unit, is removed. Some constant source B_1 is chosen and placed at a fixed, convenient position with respect to the surface S_2, one of the pair S_1 and S_2. The illumination thus obtained is taken as unity. Next, two sources A_1 and A_1' are found and placed on a cone whose axis is perpendicular to S_1, so that either alone makes the illumination on S_1 equal to that on S_2 under B_1, and gives therefore unit illumination on S_1. Now letting A_1 and A_1' illuminate S_1 together, and extinguishing B_1, a source B_2 is found such that, placed with respect to S_2, it makes the illumination on S_2 equal to that on S_1 under both A_1 and A_1'. A source A_2 is found and placed on the cone so that the illumination of S_1 under A_2 is equal to that of S_2 under B_2. This illumination on S_1 is two and it now has a clear physical meaning to speak of this illumination as *twice* that under A_1. This method of assigning numbers can be extended indefinitely for integral as well as fractional values, so illustrating the last axiom. To define fractional values we must find n sources placed in such positions on the cone, that any one of them makes the illumination on S_1 equal to that on S_2 under a constant source, and such that all n together make the illumination on S_1 equal to a unit illumination. Each of these n sources gives an illumination equal to $\frac{1}{n}$ of the unit illumination.

The construction of such a series of standards, both integral and fractional, in terms of explicit physical operations which take note of qualitative homogeneities and differences, is the logical prius to any other mode of physical measurement. Only now can the theorem, that illumination is inversely proportional to the square of the distance from the source, be given experimental confirmation. Only after the physical meaning of numerical operations has been thus fixed, may mathematical variables be introduced to denote unambiguously portions of subject matter. Only then may the movements of pointers be taken as signs for qualitative differences.

Clearly, the most important of the operations used in the definition of magnitude, are those fixing the meaning of addition. Operations for defining physical addition can be found for mass, length, period, electrical resistance, area, volume, force, and about a dozen others; for these, processes of fundamental measurement can be found, while other properties studied in physics are measurable only in terms of them, that is, derivatively. The example chosen

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[14]Walsh, *Photometry*, p. 7. Campbell points out that heat is capable of addition in a very restricted sense only, because no definition of addition will make the commutative and distributive axioms verifiable. *Physics*. p. 287.

makes clear, as Helmholtz pointed out long ago,[15] how experimental a concept addition is, valid only in so far as axioms 9, 10, and 11 are verified. It should, moreover, help expose the dogma that measurement consists in the observation of pointer coincidences, as well as the belief that addition is exclusively spatial juxtaposition and division. Undoubtedly, spatial juxtaposition is the primitive meaning in the addition of lengths; but in the example used addition involves *conjoint activity* of sources; in measurement of weights, it means the establishment of *rigid connections* between solids; in the estimation of time periods, it requires the *temporal repetition* of certain rhythms; in the evaluation of volumes it signifies the discovery of liquids which *fill containers* without implying spatial contiguity. The presence of spatial relations is not a sufficient condition for addition; there is necessary a distinctive qualitative context, inclusion in which identifies different instances of addition, as addition of the *same* characters.

When once the standard sets of different magnitudes are constructed, the measurement of even those properties whose standards these sets are, is indirect, and consists in the comparison of the properties with their standards. The comparisons are very often circuitous, because advantage is taken of the relations between variations in the fundamentally measured magnitudes and the variations in the position of a pointer with which they are in some physical connection. It is easier to make the correlation between pointers and weights once for all, than to engage in fundamental measurement whenever one is estimating the tonnage of an elephant.

IV

Some objections examined. The preceding analysis and exposition (1) assumed without much question that measurement was the evaluation of the empirical relations between physical objects diversely qualified; and (2) stressed the importance of the distinctions between extensive and intensive qualities. Both of these doctrines have met some opposition.

(1) Impressed, no doubt, by the profound difference in cognitive status between the unmeasured and measured qualitative world, Russell converts that difference, achieved in terms of the processes already examined, into a difference between a concrete actuality and a realm of essences; this latter realm is understood not as the ordered relations of and between existences, but as a domain of immaterial entities having no necessary reference to existence. For Russell, therefore, actual footrules are quantity, their lengths are magnitudes. It is only by an ellipsis that two quantities can be said to be equal: they are equal because they possess the same magnitude; and it is improper to say that one of two quantities is greater than the other: what is meant is that the magnitude which the first quantity possesses is greater than the second magnitude. Only quantities can be said to be equal, by having the same magnitude; two magnitudes cannot be equal, since there is only one of each kind.[16]

Russell's first objection to the relative view of quantity (the view that $>$, $<$, and $=$, are relations holding directly between physical things), and his espousal of the absolute theory (for which things must first be referred to an otherwise

[15] Helmholtz, *Op. cit.* p. 89 ff.

[16] *Principles of Mathematics*, p. 164 ff.

undefined realm of magnitudes in order to be measured), is based upon the observation that in any proposition asserting $>$, $<$, or $=$, an *equal* quantity may always be substituted anywhere, without altering the truth value of the proposition. It is not the *actual* quantity, but some character which it has with other quantities that is of importance.

It will be granted that this point is well taken, if possession of relations like equality is interpreted without reference to the specific process which defines equality. When two weights are equal, that relation holds by virtue of the way the two weights enter into a complicated evaluating process. Two weights are not equal "in themselves", they are equal as a consequence of the manipulation to which they are subjected. Of course, once having specified the defining operation, whether it is actually performed or not, the things measured have a nature prior to the actual performance which conditions their behavior in it. This observation may be verbal only: if equality is defined in terms of the process, quantities can be called equal prior to the process only proleptically; unless at *some* time the process eventuates, we cannot know that there is such a property as equality.

But it is one thing to say that relations like equality hold between two objects only in specified contexts, and another thing to convert those relations into possession of some third entity or common essence, incapable of every empirical verification. That third entity is the reification of a relation. The absolute space which haunted physics is just such a hypostatization of relations; space and time may be construed more simply as pervasive relations between events, rather than as containers, extrinsic and outside the changing qualities. So when magnitudes, which are always found to be relations exhibited in the physical operations of things, are invoked as the locus of those operations, it seems legitimate to ask what empirical difference their existence or non-existence as "common essences" would make.

Russell's second point is that every transitive symmetrical relation is analyzable into a complex of two assymetric relations and a third term, and that since equality is such a relation, it should be analyzable into the possession of a common magnitude by two quantities. "The decision between the absolute and relative theories can be made at once by appealing to a certain general principle, the principle of abstraction. Whenever a relation of which there are instances is symmetric and transitive, then the relation is not primitive, but analyzable into sameness of relation to some other term, and this common relation is such that there is only one term at most to which a given term can be so related, though many terms may be so related to a given term."[17]

If the analysis given above is sound, all measurement which is not fundamental in the sense defined, or which is not surrogative in the manner to be explained below, consists in the direct or indirect correlation of quantities to be measured with the standard series. It is correct to say, therefore, that two equal footlengths are equal because they possess a common magnitude, *if* this means that they are compared, ultimately, with the same member of a standard series. But in establishing equal magnitudes *within* the standard series itself, no reference to a "common term" was necessary. Equality, it is true, was not taken as a fundamental character, since it was defined in terms of two transitive

[17] *Op. cit.*, p. 166.

assymetrical relations. But those relations held or did not hold between *qualities,* and did not relate to some third term *outside* the qualitative series. The weights *a* and *b* are equal, because with respect to the relation "greater than" which defines the series of weights (it may be defined in terms of the sinking or rising of the arms of a lever), *a* is neither greater than nor less than *b,* and if *c* is any other *weight* (a member of the *series,* not *outside* of it) then if *a* is greater or less than *c* so also is *b*. No unique third term outside the physical domain is involved in defining equality; if Occam's razor still can cut, the magnitudes demanded by the absolute theory may be eliminated.[18]

(2) The doctrine that magnitudes are essences, and therefore not divisible or additive even though it is only between them that the relation "greater than" can hold, leads very naturally to the view that the distinction between extensive and intensive magnitudes is purely conventional. Additiveness belongs, on the absolute theory, only indirectly to magnitudes, as a manner of speaking of the addition of quantities whose magnitudes they are. For "addition" of two magnitudes yields two magnitudes, not a new magnitude, while the addition of two quantities does give a new single whole, "provided the addition is of the kind which results from logical addition by regarding classes as the wholes formed by their terms."[19] Addition, for Russell, thus always refers to the conjunction of collections, and to the enumeration of the number of parts in the new whole. And even in the logical addition of the classes there is no clear warrant, he believes, for affirming that the "divisibility" of a sum of *n* units is *n*-fold that of one unit. "We can only mean that the sum of two units contains twice as many parts, which is an arithmetical, not a quantitative judgment, and is adequate only in the case when the number of parts is finite, since in other cases the double of a number is in general equal to it. Thus even the measurement of divisibility by numbers contains an element of convention."

Now, indeed, if magnitudes express the results of physical measurement, it is not the magnitudes which are added, just as the measure of anything measured it not itself measured. If magnitudes have a logical status, then surely it is only logical addition of which they are capable. For *magnitudes,* therefore, the distinction between extensive and intensive has no *meaning.* Nevertheless, on Russell's theory, it is magnitudes which are measured, and it is because he takes "logical sum" to be the primary sense of addition that he can find only a convention in the addition of spatial distances or time intervals. How the transition from the conceptual to the existential order is effected, how "logical addition" may receive an interpretation in terms of physical operations, is a consideration omitted from his analysis. Is it not, however, more perspicacious to

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[18]Couturat declares that sound common sense sides for the absolute theory. "It is reasonable to suppose that equal magnitudes have something more in common that those magnitudes of the same kind which are not equal; and it is contradictory to suppose that two equal magnitudes are differentiated from two unequal ones only by the fact of the unessential relation which connects them: equality in one case, inequality in the others." *Rev. de Met. et de Morale,* 1904, p. 677. The reified concept is very evident in the "something in common." Moreover, on Russell's theory there is a magnitude for every specific kind: a specific magnitude of pleasure for each grade of pleasure, a specific magnitude of density for each grade of density. In the magnitude of density of two substances, whose densities are the same but are otherwise very dissimilar, *two* magnitudes or *one?* On the absolute theory, since two magnitudes are of the same kind if one can be greater or less than the other, the answer is: one magnitude. But what determines whether two magnitudes *can* be compared? Is not the decision made, not by appealing to the magnitudes, but to the physical operations on quantities?

[19]*Op. cit.* p. 178; cf. also *Analysis of Matter,* p. 116.

think of mathematical "addition" as a *universal,* whose variable empirical content will be *cases* of addition, but which will require further specific definition and experimental proof of the presence of those formal characters which make those empirical contents instances of that universal?

Nevertheless, the unusual sense in which addition is sometimes used should not be overlooked. The order generated by the relation "male ancester" may be measured in the following fashion. Suppose *a* is the father of *b, b* the father of *c, c* the father of *d;* then the "relation-distance" father- of is "equal' in all the pairs *ab, bc,* and *cd,* and *a* has the relation of "great-grand-father-of" to *d.* By an obvious convention, we could express the ancestral relation of *a* to *d* as three times the relation of *a* to *b;* and we could say that the relation of *a* to *c* is the sum of the relations of *a* to *b* and *b* to *c.* It is clear, however, that the "addition" here defined does not possess *all* the formal properties demanded by the axioms; it does not obey, for example, the commutative rule. The addition here defined is not very much more than the ordinal arrangement of relations. It is the failure to recognize the necessity of obtaining all the formal characters in fundamental measurement, which makes so unsatisfactory the attempt of Spaier to defend the measurement of non-spatial properties, and which enables him so easily to minimize the distinction between intensive and extensive qualities.[20] The introduction of numbers has a function more inclusive than the *identification* of quality.

V

Surrogative measurement. In the light of what has been said, the dichotomy between primary and secondary qualities becomes more illuminating if we view this distinction not as between objective and subjective, efficient and otiose, pervasive and local, permanent and evanescent, but as between those qualities which are capable of fundamental measurement and those which are not.

No science, certainly not physics, can dispense with qualities that are incapable of addition in the fullest sense, and the progress of modern science has consisted very largely in bringing non-additive qualities like density, temperature, hardness, viscosity, compressibility, under the sway of numerical determination.[21] There is of course an obvious way, already suggested, how this could be done. That way is to place qualities like density into a serial order, and to assign numbers to points of this qualitative spectrum. It is characteristic of modern science, however, that such is *not* the method which has been adopted, just as in zoology it is with bats and not with fish that whales are classified. For it is the particular virtue of modern science not to be concerned with the grouping of the most obvious qualities, thereby treating them as isolated from and unconnected with other groups; that virtue resides in the persistent attempt to obtain well defined connections, expressed mathematically wherever possible, between qualities measured or measurable fundamentally and those incapable of such measurement.

Unfortunately, the correlation of qualities has been often interpreted as the production of secondary qualities by the primary ones: the latter alone have been endowed with causal efficacy, the former degraded as otiose and epipheno-

[20]Spaier, *op. cit.* pp. 242-55.

[21]Duhem, *L'Evolution de la Mecanique,* p. 199 ff.

menal. Thus a distinction which in *operation* is a practical and logical one, has been converted into a distinction between grades of reality, on the ground that causes are more real than their supposed effects. Mathematical physics has been understood to make nonsense of the poet's cry—"Natur hat weder Kern noch Schaale." None the less, all that the equations of physics and the method of establishing them do imply is that non-additive qualities are inextricably interwoven with additive ones. This dependence is existentially mutual; from the point of view of the *logic* of measurement, it is assymetrical. It is because of this dependence, expressible in the form of numerical laws, that numbers may be correlated unambiguously with non-additive qualities; it is because of such laws for density that of the four sets of numbers entertained on a previous page, only the last set is adopted.[22]

How non-additive qualities may be unambiguously denoted will be clear from an example. In the case of density it is discovered that the weight of an object is intimately connected with its volume, a connection exhibited in the uniform association of these characters. Weight and volume are measurable fundamentally, so that independently of the numerical relations which affirm their uniform association, numbers can be assigned to them. There is no special theoretical difficulty, although there may be many practical ones, of determining the value of the constant which expresses this uniform association and which mathematically is the ratio of the numerical value of the mass to the numerical value of the volume. The *form* which the mathematical equation will take is, of course, dictated only partly by the measurements on the properties studied in any *one* instance: more general considerations will come into play arising from the desire to make the many numerical equations themselves interconnected and parts of a unified doctrine.

In most cases, the order of the constants or ratios which are determined for several objects can be shown to be the same order as the serial order of some non-additive quality which the objects possess in addition to those already measured fundamentally. So the order of the ratios of mass to volume is identically the order of the density of liquids as defined by their floating capacities relative to one another. Consequently, the same set of numbers may be used to denote both the uniform association of mass and volume and the relative buoyancies. It goes without saying that the numbers thus obtained for qualities from numerical equations are not always amenable to a physical interpretation of addition. When one body is said to be thirteen times as heavy as another, a meaning must be given to such a statement different from the meaning of the statement that mercury is thirteen times as dense as water; only in terms of the numerical law connecting mass and volume has the latter proposition significance.

All equations which define a constant, to be identified perhaps with some property capable of definition independently of the equation, require, therefore, that the other terms of the equation be measurable without reference to the defining equations of the constant. In the sense that the constants are defined, not *all* magnitudes can be defined without circularity; there must be an ultimate

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[22]"The ascertainment of qualitative features and relations is called measuring. We measure e.g. the length of different chords that have been put into a state of vibration, with an eye to the qualitative difference of the tones caused by their vibration, corresponding to this difference of length." Hegel's *Logic* (Wallace tr.) p. 200.

reference to magnitudes obtained by fundamental measurement. The equation $pv=RT$ has meaning only if p, v, T have a meaning outside of this equation; only then may R be determined experimentally. If p, v, T are not measurable fundamentally there must be a chain of equations connecting them with magnitudes which are. It is a testimony to the endless *complexity* of nature, not to her poverty of qualities, that only six independent fundamentally measured magnitudes are required for the investigations of physics. "There are only a few independent magnitudes in physics. But between these and the countless number of independent magnitudes appearing in human life, there is no sharp separation." Strength of wind would be a genuine aspect of some events, even if strength of wind were completely definable, which it is not yet, in terms of the velocity and force of air particles.[23]

Moreover, the power of all symbols, and especially of numerical symbols, to refer simultaneously to several contexts must be recognized if mystification is to be avoided. The unification and identification in statement that follows from the introduction of mathematical methods is due to the pervasiveness of certain formal characters in situations qualitatively different, which are, as a consequence, capable of a unified treatment. In the interpretation of equations as the literal identification of different qualitative continua, and as the attribution of intrinsic, non-relational common characters to diverse subject matter, lies the force of most of the petulant criticism of science. Let us, for example, study the equilibrium conditions of a beam balance. If x, y, z represent certain bodies A, B, C, then the equation $x+y=z$ will mean that A and B on the same pan will balance C on the other, and it will also mean that the numerical measure of the weight of C is equal to the sum of the measures of A and B. The perhaps less relevant case of a chemical equation like $2H_2+0_2=2H_20$ represents theoretical conceptions like atom, valence, physical operations like passing a spark through a mixture of two gases, and numerical relations between the measures of weights and volumes.

The confusions which arise from the failure to note how complex the functions of symbols may be is illustrated in a recent criticism of the achievements of science. The principle of the lever, as expressed in the form of a proportion, is the climax of an extended critique, but this special discussion is reproduced in full.

"It will be best to point out that it is the result of two apparently unjustified leaps of imagination. . . A double leap it is, and the reader can supply whatever theory of revelation, reincarnation, or conventional fiction he prefers to account for it.

"To say that W_1: W_2:: D_2: D_1, is by itself ambiguous. Perhaps it only means that certain numbers stand to each other this way: 2:4 :: 3:6. In that case it is merely a happy discovery in arithmetic. But W stands for weight and D stands for distance. It may, therefore, mean that the relation between two weights is the same as the relation between two distances. But this is not true for many relations; for instance, 'heavier than' is a relation between two weights, but not between two distances. The only relation that works is a hybrid combination of these two.

"The combination is evidently derived from two previous proportions,

[23] *Cf. Handbuch d. Physik,* Bd. II, p. 5.

namely: W_1: W_2:: 2:4 and D_2: D_1:: 3:6. Then because we already know in arithmetic that 2:4 :: 3:6 we can finally see how it is that W_1: W_2:: D_2: D_1. But why weights and distances are like numbers, to use the simile, has still to be explained. The only answer that I know is that some poet of the commonplace was playing with words, and somebody took him literally."[24]

It cannot be pretended that the actual history of the principle of the lever is being reported. As a deduction or validation of an important numerical law, however, it merits a conspicuous place in some future Budget of Paradoxes. The difficulties which are raised seem to arise partly from a dogma that numbers cannot be the numbers of anything without losing caste, and partly from the failure to realize that in the statement of the law at least two relations are symbolized. In the first place, the relation W_1: W_2:: 2:4 can be intelligibly interpreted as meaning that four weights each equal to W_1 are equal to two weights each equal to W_2, where "equal" is defined in some unambiguous way. Secondly, it must be observed that the *symbol* :: stands for both a *numerical* relation between numbers assigned in the way suggested, and a *physical* relation into which the lever enters in a very specific way. The proportion as it stands expresses *two* sets of relations, and we can use the same *symbol* to represent both because the relations have certain formal properties which are *identical.* Numbers are not *like* weights, indeed, but numbers are, and numerals express, definite properties of weights. It is a great sin to *compare* the statement of a relation with the relation itself. If discourse cannot be literally compared with what it is about, is it not wisdom to recognize that discourse expresses it? The equation, in every case, is a symbolic statement, pointing to several aspects of the subject matter. When once the plural referents of the symbols are made explicit, and when numbers are not regarded either as common qualities or as chaste platonic beings, but rather as the expression of relations or operations between qualities, belief in the power and validity of mathematical physics need not be superstition.

As there are critics who find the application of numbers to additive properties a puzzle, so there are other critics who challenge the validity of the application of numbers to non-additive ones. It is never the non-additive qualities that are measured, it is said; velocity has a unique existential quale as a velocity, and to express it as the ratio of space and time is to measure space and to measure time, but it is not to measure velocity. A twofold reply may be made.

(1) There are many qualities which, as a matter of practice, are measured as derived magnitudes by means of numerical equations, but which can be measured fundamentally. Bridgman has shown how this may be done for velocity. Areas, volumes, or electric charges, are in the same position: they are usually measured as derived magnitudes, but are capable of fundamental evaluation. Nevertheless, even when they enter numerical equations derivatively and measured therefore in their relation to other characters, *they* are measured.

(2) It is true that most qualities, being incapable of addition, must be measured in terms of their "surrogates" in numerical equations. If the term "measurement" is restricted only to such qualities which are fundamentally measurable, it must be acknowledged that density and acceleration are incapable

[24]Buchanan, *Poetry and Mathematics,* p. 90.

of measurement. But by calling the process of assigning numerical values to density some other name, the *significance* of what is done is not destroyed. What is beyond much doubt is that the measure numbers of those characters incapable of addition, or even incapable of sensuous intuition like magnetism, are not mere numerals or formulae for nominal combinations of additive characters; they represent rather certain coordinated qualities or certain relational properties of the systems studied.[25] If it is the expansion of mercury that we actually measure in the more restricted sense, the measurement is performed because there is a uniform association between this expansion and qualitative temperature changes.[26]

Several modes of surrogative measurement may be obtainable for the same property. So we may define the temperatures of black bodies by using the well-known Stefan-Bolzmann law of energy radiation. Two temperatures will then be equal, if the energy of radiation which the surfaces of black bodies send out in a given time into a given space, is the same for the two temperatures; two temperature differences will be equal, if the differences of energies radiated are the same. Except for the zero point and unity, the temperature scale is defined. Comparison of this scale with the gas thermometer scale would show that the scales cannot be made directly congruent. However, by a proper choice of the zero point, the numbers of the gas thermometer are found proportional to the fourth roots of the numbers of the energy scale; if the fourth roots of the energy scale are used to define the temperature of the black body, the two scales can be used interchangeably.[27] Similarly, velocity may be measured not in terms of space and time, but in terms of the resistance which a given body meets when moving through a specified medium.

It is by discovering the recurrence of certain constants in different numerical laws that the ideal of a unified science is progressively realized.

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[25]Meinong, *op. cit.* p. 228, 275. The expression "surrogative measurement" is, of course due to Meinong.

[26]But one need not therefore confuse, as does Hegel, the distinction between temperature as intensive and the correlated expansion, which is extensive. Hegel, *op. cit.* p. 194.

[27]Runge, *op. cit.* p. 8.

CHAPTER III

THE SPECTROSCOPE—AN HISTORICAL EXCURSION

I

The child who is undecided whether to spend or save the few cents it has obtained illustrates in a very obvious way the conflict between two types of ethical doctrines. There are doctrines which, representing an uncivilized sensualism, find present enjoyments alone valuable, and there are others which, championing an inhuman economy, sacrifice continually at the altar of the future. This opposition has its analogue wherever reflective thought seeks the *evidential* value, the *order* of succession or coexistence, of immediately noted subject matter. It finds a parallel in some conflicting interpretations of the nature of scientific method and scientific objects: there are interpretations which claim for immediately apprehended qualities exclusive reality and explicit cognitive status; and there are other interpretations which postpone indefinitely the acknowledgment of the compulsion which a presented reality exercises upon our intellectual formulations.

The progress of physics toward abstractness and generality, the consequent intertwining of the immediate and the mediate, has been sketched in general terms in the preceding chapter. This development can well bear a more concrete illustration than the one already given, and in the present chapter, therefore, one instance of that growth will be briefly surveyed again, but historically and not logically. In contemporary physical investigations the outstanding unifying and heuristic concept is that of the quantum. Its introduction by Planck has been noted by philosophic critics of science as a revolutionaly innovation, as a revival of Pythagoreanism, as a sign that science has reached the limits of its powers, as the definite abandonment of the physics of continuous media. About the ultimate interpretation of the quantum physicists themselves are not of one mind, and only highly specialized investigators within the already specialized domain of physics have a comprehensive knowledge of its far-reaching applications. If, however, the evaluation of the present status of the quantum is not a task for the non-physicist, one may ask with some hope of a satisfactory answer for an outline of the chain of evidence which led to its introduction in 1900 as the outcome of studies on black-body radiation. By observing, so to speak, the quantum in action at its very birth, when still unencumbered by the more recent developments of physics, its special meaning and function may be clarified and the role of such concepts illustrated. It is such an outline of *some* of the relevant considerations which lead from familiar qualitative events to their highly abstract formulation in atomic physics that is attempted in the second section of this chapter.

The role of the instrument in physics has been honored more in the practice of the laboratory than in most philosophic formulations of scientific method.[1] Scientific instruments are caused to be born primarily to supplement the discriminative

[1]But see Hook, *Metaphysics of Pragmatism,* Chap. I; and Duhem, *L'Evolution de la Mecanique,* pp. 209 ff.

powers of human sense organs, to extend the domain of possible investigation, and to serve as standards of uniformity not attainable without them. It is an *instrument* which makes possible the fine distinctions between the weights of lead obtained from different sources, or the estimation of terrestrial and celestial distances. Judgments by unaided eye and ear are found to be crude and diverging, and are eagerly replaced by instruments more sensitive to complicated physical changes and more converging in the evaluations they report.

But this initial function is often overshadowed by the role instruments play, first, as middle terms or connecting ties in the unification of various fields of inquiry, and, secondly, in the application of theory to gross subject matter. The rotation of the earth which marks equal time periods also serves as a link between the study of tides and the origin of the solar system. The thermopile which makes possible sensitive temperature measurements also is a means of connecting electric and caloric phenomena. And a complicated instrument like the spectroscope is the point of intersection of chemical, optical and mechanical and thermodynamic investigations. Furthermore, it is the instrument which translates the symbols of abstract theory into clearly demarcated qualities of subject matter. For physical *theory* is a series of deductions from first principles entertained hypothetically: as a deductive system it is a branch of pure mathematics, and *somewhere*, therefore, the symbolic terms and relations of the theory must be equated to properties of a physical system. A complete philosophy of mathematics is, fortunately, not necessary in order to recognize the difference in the procedure of the mathematician and the experimental physicist. It is, however, some instrument, whether an eye, a hand, a telescope, a microscope, or a galvanometer, which bridges the gap between the subsistential realm where mathematics is so often located, and the qualitative manifolds which constitute the ostensible subject matter of physical science.

Nevertheless, the layman at any rate meets almost insuperable difficulties when he attempts to locate somewhat exhaustively the existential correlates of that conceptual structure known as theory, or to discover the qualitative situation symbolized by the calibrations and pointer readings of most instruments actually used to translate theoretical distinctions into existential traits. For the use of instruments involves a host of special theoretical studies, in terms of which the obvious manipulations of their parts become signs for absent and surprising characters. An instrument, like a person, has an obvious appearance, and one is sure to be wrong if as in character reading, appearance is taken as a certain sign of subsequent behavior. The specific shape and arrangement of the parts of an instrument have for their background much implicit theoretical and experimental material, so that the explicit behavior of the instrument rarely points directly to the correlated and inferred qualitative manifolds. Something of this has already been encountered in the various modes of surrogative measurement of the same physical property. In embarking upon the study of a single instrument one usually invites oneself to a study of much special theory and of many other instruments, in terms of which the function of the first one becomes intelligible. In science, at least, experiments and propositions isolated from a body of other experiments and propositions have little value. Every instrument is only a link in the endless chain which constitutes our knowledge of the physical world. Every instrument has precipitated around it a compli-

cated network of theories which alone give the instrument a representative, mediating office.

Consequently in any attempt to trace the relevance of the quantum concept to the operations performed and distinctions noted in the immediate qualitative world, the instrument most intimately associated with the quantum must be the principal object of one's study. It is, therefore, the spectroscope which must be studied, and studied with an eye to both its correlating and translating function. But since because of the fearful complexity of the instrument, only a schematic survey of the role of the spectroscope is undertaken here, this study can perhaps be best made by tracing the high points of its development up to 1900, and so indicating the circumstances which attended Planck's far-reaching innovation of that year.

In this sketch of the development of the spectroscope, the aim will be to illustrate the thesis that progress in physics consists in an advance in the *generality of formulation* of directly observable events. That generality is won by turning away from the enjoyment of the qualitative wealth of individualized events, and by seeking to discover connections between certain qualities *deliberately* isolated from their context. The progress of physics never carries the student away from the world of familiar experience into a world that is as substantial as that world, but completely unlike it. The world of physics *is* the qualitative world in its co-existential and temporal order. It is not another world in addition to the qualitative world. But this generality of formulation is won at the price that it is not an exhaustive account of any *one* member of a class of phenomena, although it is applicable to *all* members of that class. Consequently, the generally is won by passing from the events immediately observable to *relations* between them, and to relations between these relations.

Because spectroscopic phenomena are very numerous and complex, general principles are demanded by means of which the otherwise indigestible mass of measurements obtained with this instrument can be coordinated and the direction for new research indicated. It is for this reason that the spectroscope has played varied roles in the history of physics. The first and early function of the spectroscope was the evaluation of constants, associated with the colored lines of the spectrum, and interpreted as wave lengths in a periodic process. Then came the identification of chemical elements and compounds by their observeable effects on the spectrum. Finally, the study of the heating effects of the lines, measured in terms of energy units applicable to a variety of energy forms, and the discovery of the order of their succession, became the chief preoccupation of spectrosopists. Data for interpretation accumulated rapidly, and before long, as will be seen in outline, the spectrum became evidence for the behavior of submicroscopic, unobservable objects like atoms and electrons.

It may be urged, therefore, that if unobservable entities are discoverable by means of the spectroscope, this instrument is an instrument for the discovery *not* of the relations between the events in the common-day world, but of a different level of reality to which, in fact, the events in the common-day world are reducible. Is not the discovery of atoms, electrons, etc., a challenge to the thesis that physics accomplishes only a formulation of the common-day world and never leaves it? To this challenge the remainder of this section is briefly addressed.

Certain undeniable admissions must be made. The functions of any theory

are manifold: It may satisfactorily correlate information about a portion of nature already well known, just as the undulatory theory of light accounted for the familiar events of reflection, refraction, and polarization. It may be a fruitful principle of research for predicting and developing unexplored segments of nature, just as the wave theory predicted diffraction phenomena. Or it may be an intellectual instrument for unifying domains of investigation originally regarded as disparate, as light and heat phenomena were coordinated into a single theory by the wave theory. But in every theory and in each of its functions, as indeed in all discourse, what is immediately presented perception is the point of *departure* and not the goal of inquiry. The "facts" of every-day experience no less than the abstractions of physics are the residue of a long, more or less explicit, intellectual process which takes discriminated elements of sense as *signs* for a non-present occurrence: the hands of a clock do not, by their mere presence, announce their time-telling function, any more than the luminous points in the sky directly tell that they are stars; the discovery of the significative value of the weather vane, no less than that of the spectrum, rests upon inferences and assumptions which can be tested only discursively. If nature withholds her secrets from men, it is because her language must first be learned.

An argument for the existence of atoms is, therefore, not ruled out of court because an inferential process is involved in demonstrating such existences. The claim for an existential status for sub-microscopic entities that is in no way different from the existential status of tables and chairs, is not invalid merely because such entities cannot become objects of a direct commercial exchange or of a direct perception. The existence of atoms is no more disproved because they cannot be apprehended by sense, than is the existence of our own brains, or the other side of the moon, or the erstwhile existence of some historical character in whom we believe. Proponents of the existence of atoms, in the sense indicated, argue, and argue rightly, that if certain assumptions are made about immediately observable events, the grounds for believing in the existence of atoms are no weaker than the grounds for the past existence of Julius Caesar. The validity of the claim in either case is something that must be established by competent judges for the particular claim in question, and cannot be dismissed, without an abject surrender of all theory and all discourse, on the grounds of a positivism such as that of Comte and Mach.

Nevertheless, some hesitation in affirming the substantial existence of atoms may be justifiable on other grounds. Physical theories have repeatedly oscillated from one to another of two types of procedure, sometimes contrasted as the phenomenological and the atomistic. The first, embodied most completely in the Lagrangian method in mechanics, consists in the determination of the mathematical relation between the parts of an event each of which is open to direct observation, and is further illustrated by the mathematical theory of the conduction of heat, or by classical thermodynamics. The second, much cultivated by English physicists, consists in the analysis of an event in terms of other events not directly observable, but regarded as more "simple" physically; these unobservable events are based, however, upon analogies drawn from easily representable observable patterns of behavior.[2] The interpretation of thermody-

[2]For a much more adequate analysis, see M. R. Cohen, *Journal of Philosophy*, Vol. XV, p. 365 ff.

namical properties by means of a kinetic theory of gases, is the most striking success of this method. Also, in the recent spectrum studies, the position and energy values of the different colors have been correlated with the behavior of hypothetical molecules, atoms, and electrons, whose function is to serve as models, easily visualized, in terms of which the complexity of factual detail can be coherently formulated. The brilliant success of the second method in recent years in correlating the formulations of one domain of inquiry with the formulations of another, has been the chief argument in favor of acknowledging the "physical reality" of its models.

It is important to observe, however, that although the unambiguous existence of atoms, etc., *may* in many cases be well authenticated even though never directly observable, *some* grounds for a healthy scepticism remain. In the first place, Poincaré has demonstrated that if one hidden mechanism can be devised, it is theoretically possible to discover an *indefinite* number of such hidden mechanical models for the behavior of a system of events regarded as physically complex.[3] It requires only imagination and ingenuity to find them, although it is true that most of them would be useless for extended research. And when a science is young, it often happens that the models used to formulate different sets of properties are not mutually compatible. The aim of science is, and must be, the discovery of *sufficient,* not of necessary conditions, for the occurrence of the properties it studies. Thus, the rigid solid atoms, adequate to explain the thermodynamical behavior of a substance, e.g. specific heat, conductivity, viscosity, are not adequate to explain its elastic properties; and atomic models highly successful in accounting for spectroscopic effects are not competent to explain chemical valency. If therefore, the hypothetically entertained submicroscopic entities are regarded as fully existent as a table on the ground that a sufficient condition is therewith obtained for the explanation of the occurrence of certain events; if, that is, those entities are interpreted independently of the specific analysis which thy make possible and in which they function, a three-fold counter-claim can be made: (1) The conversion of a sufficient condition into a necessary one is a formal fallacy, and can be justified only if *no* other models were possible; but, (2) *other* models may be chosen, theoretically, for exercising this explanatory function; and (3), physics does work, as a matter of fact, with models that are mutually contradictory. There is at least a presumption in favor of regarding such models *primarily,* though not necessarily exclusively, as heuristic devices.

It may be urged, moreover, that unobservable mechanisms which are found imperative or congenial for explanatory purposes at one period, may be replaced by fundamentally different physical categories. The caloric, the electric fluid, the ether, all had their day, but they did not last. The descriptive or "phenomenological" method which has been reintroduced into spectrum studies by the work of Heisenberg, is a sign that the familiar Bohr model became inadequate, and that it, too, may go the way of the caloric. What will happen when

[3]See the preface to *Electricité et Optique.* The demonstration consists in showing that the number of equations which connect the coordinates and masses of the hypothetical model with the parameters observable in experiment is greater than the number of such paramenters. Consequently, the coordinates and masses can be chosen *ad libitum* and constrained to obey any law consistent with the equations.

the winds of doctrine shift to a continuous theory of matter?[4] Since the evidence for the existence of atoms has often a probability which is, objectively, not very high, and depends in part on a temporary inability to work out an alternative theory, it is wise to remain flexible to future innovations by not regarding as indisputable what in fact may only be probable.

And finally, it may be admitted that the unambiguous, individual existence of atoms, etc., can be granted, since there is much indeed which makes the inference to their existence difficult to avoid. The grounds for inferring atomicity on a sub-microscopic level are very strong, and few who have examined even a portion of the evidence are in a position to reject the conclusions to which modern physics points. Nevertheless, if we attend to the *operational* function which such possibly existent entities possess in the structure of science, we discover that primarily (because most importantly from the point of view of the actual procedure of physics) that function is to *formulate* more concisely and simply, the events of the common-day world. The atoms and electrons, although they *may* exist as chairs do, never function in science in the same way that chairs and other instruments function. They function as *intellectual* tools. *Those* atomic models are chosen out of the class of possible choices, which make a general formulation of a variety of apparently disconnected events possible in terms of a single type of behavior pattern. Since the existence of atoms and other such entities is never directly verifiable in the same way that the existence of chairs and tables is, their existence in this sense is and remains hypothetical. But their existence as indicating patterns of behavior in the common-day world is undeniable and repeatedly verifiable. "The gods are demonstrable only as hypotheses," Santayana remarks somewhere, "but as hypotheses they are not gods." As short hand symbols for such observable behavior they facilitate the expression of observable subject matter. Such a scepticism regarding the *substantial* existence of sub-microscopic entities will therefore conclude with Nunn, that "the real achievement of science is not to have disclosed any reality behind the veil of sensible things, but to have greatly extended and deepened and rationalized the scheme of the world revealed in perception."[5]

II

In this second section an attempt will be made to fulfill the promise made in the first, and the conclusions which were there indicated will be given a detailed support drawn from the technical development of the spectroscopic theory.

(a) *The discovery of "colored rays."* The spectroscope had its birth as an instrument of research in the interpretation given by Newton to the colored spectrum obtained by passing sunlight through a triangular glass prism. Before his time, although prisms were much valued as delightful toys and although many students found in them occasions for a speciously answered challenge to their color theories, the spectrum was not taken for the evidential value it might have for an understanding of the mechanism of light.[6] It was Newton who first

[4]Cf. Lohr, *Atomismus u. Kontinuitätstheorie,* p. 49 ff.
[5]*Aristotelian Proc.* Vol. 24, p. 18.
[6]Goethe, *Beiträge zur Optik* § 33.

saw in the sun's spectrum, even though there were some ambiguous anticipations, grounds for believing that white light, prior to its passage through the prism, was not physically simple. He indicated the evidence to show that it consisted of a manifold of "colored" rays each differently refrangible, (i.e. bent toward the perpendicular to the glass surface in greater or in less degree when it entered the glass from the air) but so superposed as to produce the impression of white upon the perceiver. (In truth, however, colors whether white or not, were not, for Newton, colors of the rays of light, which were to be characterized only by their refrangibility.) The function of the prism, according to him, was to *separate spatially* these rays actually contained in the sun's light, not to create them, as it is the function of a sieve to mechanically sort out grains of different sizes actually contained in the unassorted collection.

How radically new Newton's theory of light was, can be seen only against the background of the color theories traditional in his time. Everyday color phenomena, like the rainbow, the dawn and sunset, or the change in the colors of familiar objects when brought into unfamiliar illumination, must have called the attention of men rather early to a series of events which present *problems* of considerable difficulty as well as a subject matter for aesthetic appreciation. What is the relation of these events to other events like clouds, rain, atmosphere and sunshine? Some of these problems received adequate formulation by the Greeks, and by the middle of the Seventeenth Century scientific interest had already shifted from color as the direct object of vision to the mechanism of color propogation. The rectilinear propogation of light and its behavior under reflection and refraction were made the subject of geometric optics, a science so well developed by that time that only strictly mathematical difficulties remained. The one serious physical problem that still remained unsolved (due perhaps to the crude instruments which were not competent to distinguish differences in the refraction of the different colors), was the nature of the modification to which apparently homogeneous white light was subjected on passing through prisms.[7]

As long as colors were taken primarily as qualities immediately apprehended by sense perception, and studied with reference to the activity of the organ of vision, rather than as indications of physical differences not explicity defined by reference to sense organs, the study of color by physical methods could not make much headway. But already the discovery of the *camera obscura,* Keppler's studies on photometry and Snell's on refraction, had called attention to light as a process capable of investigation without taking into account its function with respect to the eye. Nevertheless, before Newton color theories were essentially, and in spite of their growing concern to formulate the occurrence of colors in terms of the mutual relations of objects, still variations upon the Aristotelian interpretation of colors as mixtures of light and darkness; two characters of events which are, however, too near the surface of the perceptible world to serve as comprehensive correlating principles. All bodies, according to Aristotle, possess in greater or less degree a "somewhat," potentially visible, called the translucent. The presence of a fiery element in a translucent medium gives light and color, its absence gives darkness. Color is defined as the boundary

[7]Rosenberger, F. *Isaac Newton u. seine Physikalische Principien,* p. 5 ff.; Mach, *Physical Optics,* Chap. 2, 3, and 4.

of the translucent contained in bodies having a determinate size and shape; it is defined to be as either *at* the boundary or as itself the boundary. Accordingly, color is produced by the intermixture of light and darkness. Light is the color of the translucent when it is realized into full translucency by fire or other heavenly substances; but when the translucent is present only potentially, there is darkness and a *capacity* for receiving color. Light is not fire, it is not a body or the emanation from a body; it is the translucent in motion or actuality. Consequently, light as such is pure white and colorless; color is the product of the incomplete actualization of the translucent: it characterizes the intermingling of the celestial white with the darkness of the translucent not in activity.[8]

Such a theory of color does not lend itself easily to exact quantitative manipulation, or to a generalization in terms of which other phenomena can be stated. For light and darkness are not qualities which lend themselves to become objects of a converging series of judgments upon them; and they are too much touched by the emotional associations connected with human purposes to be available for the subtle elaboration which some recondite optical phenomena require. Nevertheless, for many centuries variations of it helped to explain the well-known facts involving color. The deep red of the sun and stars when near the horizon was accounted for as the mixture of light with the darkness of the earth's vapor. The redness of wood fires was explained as due to the presence of smoke. Even the prismatic spectrum was interpreted on the basis of this theory. Thus, Antonius de Dominis found that solar rays which pass through the thin end of the prism become red, those rays passing through the middle of the glass appear green, while the rays going through the thickest part of the prism and mixed therefore with most "darkness" are violet. His explanation of the rainbow was along similar lines. Isaac Voss defended the incorporeality of light, and identified the cause of color with the degree of combustion of sulphur, which thus played the role of the translucent. And Marcus Marci, well nigh anticipating Newton's theory, still attributed the prismatic colors to a condensation of light in passing through the apparatus.[9]

In these speculations the Aristotelian tradition was continued which assigned to media, like prisms, a creative agency in the actualization of the colors. According to it, when a color became manifest, a *new* quality was produced whose existential status was as unambiguous as that of the instrument which was an indispensable factor in its realization. But such facts as the non-appearance of colors when sunlight passes through a plate glass with parallel faces, or the phenomenon of secondary rainbows, were serious difficulties in the way of these crudely formulated theories of light. However, Newton's many and elaborate experiments on the spectrum seem quite pointless unless seen to be directed against the light-darkness mixture theory.

Newton's optical studies grew out of his attempts to improve the telescope, and it was his (premature) despair of constructing lenses free from chromatic abberation which led him to the innovation of the reflecting telescope. In his first communication to the Royal Society concerning his theory of light, he observed that what induced him to speculate about it, was the fact that when

[8]*De Sensu,* Chap. III, and *De Anima,* Bk. II, Chap. 7.

[9]Goethe, *Geschichte d. Farbenlehre,* Fünfte Abteilung; Rosenberger, *op. cit.* pp. 15 ff.; and Wilde, *Gesch. d. Optik,* Vol. I, pp. 180 ff. and 312 ff.

sunlight passed through a prism, the spectrum was of an oblong shape, although according to the accepted laws of refraction and the theory that sunlight was physically simple it should have been circular. He convinced himself that the elongation of the spectrum in the ratio of five to one, was not due to the varying thickness of the prism or the presence of shadows. After dismissing other possible interpretations of his experiments, such as the possibility that light rays curved upon passage through a prism, he was led to what he declared was an "Experimentum Crucis" for demonstrating his supposition that "white" light was an aggregate of "colored" rays.

Newton took two boards, placed one behind the prism fixed at the window so that the beam of light which had passed through the prism would pass through a hole in this board, and the other board be placed at such a distance from the first that the incident light would fall upon the second and pass through a hole in it also. Another prism was placed behind the second board, so that the light passing through the two boards would be again refracted by this prism before it reached the wall. But turning the first prism slowly on its axis, the second board transmitted different parts of the image cast upon it, to be refracted again by the second prism to different places on the wall. "And I saw by the variation of the places, that the light tending to that end of the Image, toward which the refraction of the first Prisme was made, did in the second Prisme suffer a Refraction considerably greater than the light tending to the other end. And so the true cause of the length of that Image was detected to be no other, than that *Light* consists of *Rays differently refrangible,* which, without any respect to a difference in their incidence, were, according to the degrees of refrangibility, transmitted toward divers parts of the wall." "Light is not similar, or homogeneal, but consists of difform Rays, some of which are more refrangible than others: So that of those, which are alike incident on the same medium, some shall be more refracted than others, and that not by any virtue of the glass, or other external cause, but from a predisposition, which every particular Ray hath to suffer a particular degree of Refraction."

Newton could therefore conclude that color is not a modification of white light due to the action of the prism. Neither the color nor the degree of refrangibility of each kind of ray is changed by refraction, reflection, or any other cause. Facts which *seemed* to contradict this theory of the immutability of color, he could also explain on the basis of it. "Seeming transformations of colours may be made, where there is any mixture of divers sorts of Rays. For in such mixtures, the component colours appear not, but, by their mutual allaying each other, constitute a middling colour. And therefore, if by refraction the difform Rays, latent in such a mixture, be separated, there shall emerge colours different from the colour of the composition. Which colours are not New generated, but only made Apparent by being parted; for if they be again entirely mix't and blended together, they will again compose that colour, which they did before separation. And for the same reason. Transmutations made by the convening of diverse colours are not real; for when the difform Rays are again severed, they will exhibit the very same colours, which they did before they entered the composition."

He concluded this communication by an argument for the substantiality and corporeality of light. It was a view which he held until the end, and

which cost him dear in the controversies to which it led. "It can no longer be disputed," he declared, "whether there be colours in the dark, nor whether they be qualities of the objects we see, no nor perhaps, whether Light be a Body. For, since colours are the *qualities* of Light, having its Rays for their entire and immediate subject, how can we think these Rays *qualities* also, unless one quality may be the subject of and sustain another; which in effect is to call it Substance."[10] So completely was the break with Aristotle made. Light became the name not for a quality directly perceivable by the eyes, but for a material substance forever hidden from them.

The detailed and elaborate defense of his theory of light was carried out by Newton thirty years later, after further opposition from Hooke was made impossible by the latter's death. The elegance of his presentation and the ingenuity of the experimental details must still impress and delight every reader of the *Opticks*. What is important in Newton's work from the point of view of this study, however, is that by definitely searching for properties of the color-producing mechanism definable in relational, non-sensory terms, and by discovering *measurable correlates* of color, i.e. differently refrangible rays, he made possible the introduction of quantitative methods into a domain not so manipulable previously. He opened the door for the application of generalized, mathematical relations which could include the specifically optical subject matter as only one instance of a more pervasive type of interconnections between events.[11] The development of optics, to which Newton's work was a most significant addition, has culminated in recent times in a unified treatment of optical, mechanical, electrical, and thermodynamical properties.

But the significance of Newton's innovations, as contributions toward the formulation of such a comprehensive survey of nature, matured only slowly. The enduring qualities of his work soon became overrun by his own and his followers' bad philosophy. It is of some interest, however, before passing on to the next important development in the function of the spectroscope, to note the kind of opposition Newton had to meet, and to see how firmly imbedded the ancient theory of the generation of colors was in the contemporary speculation. Robert Hooke's official critique of Newton's communication to the Royal Society contains the essence of these oppositions. "All the experiments and observations I have made, nay, even those very experiments which he allegeth, do seem to me to prove, that *white* is nothing but a pulse or motion, propogated through an

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[10] *Philos. Transactions*, Vol. II, No. 80 (1671/2).

[11] That thereby Newton denied to colours, as specific qualities, as the objects of vision, a place in the perceived world, does not minimize the importance of that accomplishment. He says, "If at any time I speak of light and rays as colored or endued with Colours, I would be understood to speak not philosophically and properly, but grossly, and according to such conceptions as vulgar People in seeing all these Experiments would be apt to frame. For the rays to speak properly are not coloured. In them there is nothing else than a certain power and disposition to stir up a sensation of this or that Colour. For as sound in a Bell or musical String, or other sounding Body, is nothing but a trembling Motion, and in the Air nothing but that Motion propogated by the Object, and in the Sensorium 'tis a sense of that Motion under the form of sound; so Colours in the Object are nothing but a disposition to reflect this or that sort of rays more copiously than the rest; in the rays they are nothing but their dispositions to propogate this or that Motion into a Sensorium, and in the Sensorium they are sensations of those Motions under the forms of Colours." *Opticks* (1704) I, p. 90 ff. Whatever plausibility this argument possesses is due, as has been repeatedly pointed out, to the confusion of the conditions and means of perception and knowledge with the perception and knowledge of those conditions and means. The fallacy in the "nothing but" argument is seldom revealed elsewhere with so much clarity. The influence of this confusion of the objects with the conditions of perception upon modern philosophy, is excellently exhibited in the charmingly written essay of Miss E. C. Graham, *Optics and Vision*.

homogeneous, uniform, and transparent medium; and that color is nothing but the disturbance of that light, by the communication of that pulse to other transparent mediums, that is, by the refraction thereof: that *whiteness* and *blackness* are nothing but the plenty or scarcity of the undisturbed rays of light: and that the two colors (than which there are not more uncompounded in nature) are nothing but the effects of a compounded pulse, or disturbed propogation of motion caused by refraction.

"That the ray of light is as it were split or rarified by refraction, is most certain . . . But why there is a necessity, that all those motions, or whatever else it be that makes colours, should be originally in the simple rays of light, I do not yet understand the necessity of, no more than that all those sounds must be in the air of the bellows, which are afterwards heard to issue from the organ pipes; or in the string, which are afterwards by different stoppings and strikings produced; which string (by the way) is a pretty representation of the shape of a refracted ray to the eye; and the manner of it may be somewhat imagined by the similitude thereof: for the ray is like the string, strained between the luminous object and the eye, and the stop or fingers is like the refracting surface, on the one side of which the string hath no motion, on the other a vibrating one. Now we may say indeed and imagine, that the rest or straightness of the string is caused by the cessation of motions, or coalition of all vibrations; and that all the vibrations are dormant in it: but yet it seems more natural to me to imagine it the other way."[12] And Hooke goes on to interpret Newton's experiments in terms of his own undulational theory.

But such objections, emanating from defenders of the rival color theory, could not hold their own against the persuasiveness of Newton's presentation. His interpretation of colors as actually and explicity contained in white light, was made the basis for more than two hundred years of successful experiment and theory, and it was not until fairly recently that a significant doubt has been raised against the validity of Newton's arguments. Indeed, modern theories as to the nature of white light revert, in essence, to a production theory of color in which the instrument (grating or prism) plays the role of manufacturer.[13] But it has required the discovery of Fourier's theorem to make reaffirmation of the older position possible; the function of the spectroscope, on the modern theory, is to impose and separate out the Fourier components corresponding to the different color frequencies only potentially present in the irregular pulse of light. On this modern version of the old mixture theory, the instrument itself interacts and significantly modifies the light phenomenon studies. Recondite or mooted theories, such as Heisenberg's principle of indeterminism, are not necessary to compel recognition of the fact that every experiment occurs *within* nature and that its instruments are subject to the influences which the events studied exercise upon them. As Lord Rayleigh remarked, the regularities which are observed in light waves that had passed through prisms or gratings may be the work of the spectroscope, and the number of bright lines observed in the spectrum is a function of the resolving power of the instrument and proves nothing about the regularities in the original light pulse.

(b) *The discovery of the invisible spectrum.* Newton's discovery of the

[12]Birch, *Hist of Royal Soc.*, Vol. III, p. 11 ff.
[13]Wood, *Physical Optics*, 2 ed. Chap. XXIII.

correlates of color which were amenable to treatment by physical methods, soon brought about a highly mathematical theory of the propogation of light. His was the most important initial step in the direction of an abstract but comprehensive formulation. The nature of the mechanism involved in that propogation was declared by him to be corpuscular, a view abandoned only after a hundred years, with the successful application of the Young-Fresnel wave theory to "interference" phenonema. But this is a side of the history of light theories with which this survey is not concerned. However, the features of periodicity hereafter associated with the light mechanism were observed as early as Grimaldi and Hooke, and Newton's explanation of the colors of thin transparent layers by "fits of reflection and transmission" was the first quantitative identification of what later became known as the wave lengths of different colors.[14]

It is surely paradoxical to talk of unseeable light if by light is meant what can be defined and noted only by the eye. However, with the postulation of a non-visible mechanism for the propogation of the visible qualities, or, better, with the discovery of the relations which those qualities bear to one another, that paradox no longer remains if *now* by light is denoted either that mechanism or that relational structure. We come to a period of physics in which the presence beyond the visible portions of the spectrum, of activities discoverable otherwise than by direct sight, are gradually recognized; and identified, *largely by virtue of the structural identity of relations exhibited,* as the effects of the same mechanism which produces the visible effects.

The next step in the progress away from the immediacy of sense qualities was taken by Wollaston, when he improved the current spectroscopic methods of observation by substituting a narrow slit parallel to the face of the prism for the round opening habitually used by Newton. Newton and his successors had seen the spectrum as a continuous band of color. But Wollaston was now able to observe certain dark lines which seemed to be separating the green and the blue spectral colors.[15] Moreover, when he used candle light for a source, a different set of prismatic colors in different proportions appeared than when the sun was the source. But the meaning of his observations escaped him completely.

The significative nature of the spectrum came to the fore very noticeably, however, when Herschel studied the heating powers of the prismatic colors. He discovered that not all the rays of the sun contribute equally to the heat which they produce. By using several thermometers repeatedly exposed in the solar spectrum, he found that some parts of the spectrum gave higher temperature readings than others, and that the colors which produced most heat were not the ones which gave the best illumination.[16] Although his findings were not final, they were the beginning of a long series of investigations which led to a unified doctrine of light and heat: a unification which would be impossible as long as light and heat are taken as direct objects of sense, since *sensibly* they are incomparable. Herschel concluded that radiant heat as well as light is refrangible and subject to the laws of dispersion, absorption, and scattering. "May not this lead us to surmise," he asks, conjecturing on the basis of a corpuscular theory, "that radiant heat consists of particles of light of a certain range of momenta,

[14]Mach, *op cit.* Chap VIII.
[15]*Phil. Transactions,* 1802, II p. 378
[16]*Phil. Transactions,* 1800, p. 263.

and which range may extend a little farther, on each side of refrangibility, than of light?" And indeed, Herschel found upon ruling equal spaces contiguous to the red end of the spectrum, that the maximum increase of temperature was beyond the visible red, while no rays falling beyond the violet end were observed to have powers either of illuminating or heating. Great improvements in the instruments of dispersion and measurement of heat energy were still necessary before an adequate statement of the heating power of solar rays could be made. But, meanwhile, Herschel refused to identify the heat and light producing agents, chiefly on the ground that nature was not in the habit of using one and the same mechanism with any two of our senses.[17]

The ultra-violet side of the spectrum was not left long unexplored, for in 1801 Ritter announced his discovery of rays in the region by noting their chemical action upon silver chloride.[18] In fact, the reducing efficacy of these invisible rays was found to be greater than that of violet light. Within two years, he had explored the entire solar spectrum with a view toward discovering the rays of maximum chemical activity, which he located in the ultra-violet.[19] It was the observation that the position of the maximal chemical, illuminating, and heating effects were not identical, which prevented for many years the acknowledgment of a single physical agency for these diverse activities. Nevertheless, the advance in the direction of a single set of principles to account for and correlate phenomena sensibly different had been considerable. The range of the effects of solar rays had been found to extend beyond *directly* observable limits, and the agency (or agencies) of transmission was seen to be subject in each case to principles, such as refraction, dispersion, reflection, hitherto regarded as *exclusively* optical.

But perhaps the most momentous increase in the powers of the spectroscope occurred through the studies of Fraunhofer on diffraction phenomena. Led to optical studies by the desire to improve the telescope, he became the first careful student of the dark lines in the solar spectrum; and the first to use diffraction gratings to make accurate determinations of wave lengths of bright lines, by using the dark lines as fixed points for mapping the spectrum. "Without this device, all spectroscopic investigations would have remained impossible," writes Kayser.[20] While the immediate consequences of Fraunhofer's innovations were not notable, the precision in spectroscopic measurements to which they gave rise finally accumulated so much material for formulation, that only a much embracing set of general principles could save the science from chaos and despair.

How subtle and unobvious the interrelations between apparently disconnected events may be, the history of physical optics makes clear with an unmistakable force. Here, on the one hand, are more or less intense colored patches; there, on the other, are luminous bodies enveloped in gases. Can a philosophy of science which concludes that the theories of science are mere agreements or conventions, give an intelligible account of why the former are taken as *signs* for the behavior of the latter? Can an epistemological theory, upon which knowledge is a matter of obtaining *sensations*, really maintain that the *perception*

[17] *Loc. Cit.*, p. 508.
[18] *Gilbert's Annalen*, Vol. 7, p. 527.
[19] *Ibid.* Vol. 12, p. 410.
[20] Winkelmann's *Handbuch d. Physik*, 2 ed. vol. 6, p. 655.

of the spectrum is tantamount to an understanding of its significative function? The history of the laborious experiments and lucky guesses which go into developing that understanding gives the lie to both.

Fraunhofer's object in making his studies was the construction of achromatic lenses. He was therefore fortunate in making the discovery that the amount of dispersion of a color depended not only on the angle of incidence of the light, but that different colors are dispersed in different amounts, varying with the nature of the dispersive media.[21] But of greater theoretical significance was his observation that when he used the sunlight instead of lamplight as a source, he found an indefinite number of more or less intense vertical lines which were less bright than the rest of the spectrum. The lines were present for all kinds of refractive media, and for all angles of incidence. Furthermore, there was an apparent constancy in the order of their distribution, and they could therefore be used for the identification of the bright lines. That these dark lines were not the boundaries of colors, as Wollaston thought, was shown when Fraunhofer discovered them in the middle of colored bands. He believed, for small reasons indeed, that these lines were due to a deficiency in the nature of sunlight, and that, consequently, they do not arise from diffraction or illusion of the senses; he found also that they were not present when lamp light was used as a source. The correct interpretation of the dark lines was not to come for another fifty years. But meanwhile, using them as reference points, he compared the spectrum of Venus with that of the sun, and *inferred* that the light of the planet was in no way different from solar light. The spectrum of Sirius was found, however, to have marked differences from the spectrum of the sun. So began the career of the spectroscope as a chemical analyst: the different chemical elements and compounds were finally to be identified by their spectroscopic effects. It was not by agreement or by virtue of a *priori* principles, that spectrum lines were finally taken as data for inferring the chemical behavior of matter, nor was it because of some immediate intuition into their symbolical value. The kind of evidence required for ideas in science, is the evidence which comes from the successful application of the operations which define them.

The introduction of gratings to produce spectra, instead of prisms, made possible, especially at the hands of Rowland much later, highly precise determinations of certain constants, which on the wave theory of light are identified with the wave lengths of the different colors. Fraunhofer found that when light passes through a very large number of sufficiently narrow slits whose distances from each other are small and the same, there is produced a spectrum due to the mutual action upon each other of the rays bent by the grating. The occurrence of these lines is predictable on the Young-Fresnel wave theory. This spectrum is similar in many essential respects to the one produced by a prism, and the fixed lines obtained by using the latter are also found in the former.

Fraunhofer obtained gratings by using silver or gold wires, and by ruling on gold foil. He permitted sunlight to enter a darkened chamber through a narrow slit, and then to fall upon the objective of a theodolite telescope so that the image of the slit was focussed on the cross hairs. When a grating was interposed between the objective of the telescope and the slit, intense colored spectra

[21] *Gilbert's Ann.* Vol. 56, p. 272 ff.

appeared symmetrically on each side of the image of the slit, and which repeated themselves and became the wider and fainter the more distant they were from the axis of symmetry.[22] The telescope could be turned so that different lines of the spectra were capable of being focussed on the cross hairs, and the angle of rotation could be measured by a divided circle. Designating this angle by Θ, the width of the intervale of the grating by g, the thickness of the individual slits by d, and $e=g+d$, Fraunhofer found that there was a relation $\Theta_n=\frac{n\omega}{e}$ (for small angles, sine Θ can be substituted for the angle Θ) expressed in Paris inches, where n takes successive integral value, (the order of the spectrum) and ω is a number different for each line within a spectrum of the same order, but recurrent for the similar lines in the spectra of different orders. The relation involving the sine of the angle could be deduced from the wave theory of light. The number ω Fraunhofer recognized as an important natural constant: "Even those who do not endorse the wave theory of light must grant that ω is a real magnitude. Whatever else one may think of this quantity, it must be granted to have this nature, that one-half of it is oppositely directed to the effects of the other half, so that if the first half coincides with the second, the effects are cancelled; while the effects are doubled if two first halves or two last halves coincide. This is shown to be basic by interference phenomena. If one thinks of this as something else than a wave having these properties, he may suit his tastes."[23] In such (surrogative) measurements of constants, constants which are interconnected within the relational pattern of a theory, and which are uniformly associated by means of the instrument with existential qualities like color, the mediating function of the instrument between the symbols of the theory and existential traits is exhibited.

(c) *The secret of the spectrum.* The next fifty years of spectroscopy were devoted to the perfection of accurate methods for determining wave lengths, and to the interpretation of the dark lines named after Fraunhofer. Nevertheless, the investigations begun by the older Herschel on the location of the heat maximum, a type of inquiry destined to play a considerable role in modern theory, were not neglected. The discovery of the interrelations of natural forces, which Schelling's philosophy of nature had suggested, was pursued with some success by many of his disciples. For example, by using a large gas thermometer more sensitive than those used by Herschel, Seebeck discovered that the prism exercised selective absorption upon the rays passing through it, and that consequently the heat maximum was a function of the prism as well as of the nature of the light used. This discovery necessitated the use of prisms of different material in studying the different portions of the spectrum, and made the spectroscope a very complicated instrument. Seebeck also found that the limits of the spectrum are not at the limits of the visible colors, but that it extends further, "even if there is perceived by the eye no color. That light is present there, is without doubt, and indeed *visible* light, however much physicists may talk, risking confusion, of unseen light rays which are not seen by us

[22] *Gilbert's Ann.* Vol. 74, p. 340 ff.
[23] *Loc. cit.* p. 369.

because of the lack of sensitivity of the eye."[24] The controversy as to the identity of light, heat, and chemically active rays was not yet over.

At the same time, the accuracy of the studies of heat distribution became much improved when Melloni substituted the thermopile, also a discovery of Seebeck, for the thermometer. This new instrument became a powerful weapon in his hands and in the hands of Forbes in the latter's work on refraction and polarization of heat. The discovery of properties in light and heat phenomena, such as those of absorption, refraction, polarization, and interference, which were identically formulable, made an irresistible demand for a single doctrine which did not treat of light and heat *per se*. The way in which such doctrines develope is therefore not difficult to see. Their acceptance does not depend upon denying the qualitative incompatibilities of immediately observed events. Their acceptance does not indicate a stratum of reality disclosed beneath the appearance of common-day experience. Observed *relations* between *visible* events and some instruments can be formulated and the *relations* between *temperature* phenomena and that instrument can be stated in terms of the same formulation. Upon the basis of the *relational* identity of two qualitatively dissimilar groups of events, a theory may develop which can, possibly, indicate *further* relational equivalences. It is the identity of certain relational patterns, not of some material substance, which the theory expresses.

The use of the thermopile added new complexities to the study of the spectra. But the principles involved in the thermopile are simple: When two metals are soldered together and the point of junction heated, an electric current passes from one metal to the other. The current may be conducted through a delicate galvanometer, the needle of which serves as an index of the increase in temperature. The deviations of the needle are measured on a divided circle, and while they are not in general proportional to the intensity of the current producing them, under a twenty degree deviation such a simple correlation can be made. In order to disclose the connection with the temperature changes, Forbes found "that the same quantity of heat falling on the photometer ball of an air thermometer and on the pile, moved the liquid of the former 1° and the needle of the galvanometer 4°.2. The degrees of the photometer being tenths of 1° Centigrade, one centigrade degree would correspond to 42° of the galvanometer."[25]

How complex the existential interconnections, and the theoretical assumptions and interpolations are in a physical experiment, is perhaps sufficiently evident from these researches of Seebeck and Forbes. But however indirect the symbolic function of pointer readings may be, the reference of the readings and the theoretical assumptions to a converging series of reports on observable qualitative events is never lacking. The galvanometer needle indicates a flow of current; which indicates a change of temperature; which indicates differences of energy transmitted by the light mechanism; which indicates differences of wave length; which indicates differences in the colors observed. The chain of inference may be very long and difficult to exhibit in all its details. Its validity, however, is always an experimental matter capable of verification or refutation.

Now came the series of studies on emission and absorption spectra by such

[24] *Berliner Akad. d. Wiss., Abhandlungen,* 1818-9, p. 346.
[25] *Edinburgh Trans.* Vol. 13, p. 138 ff.

men as J. Herschel, Brewster, Talbot, Foucalt, Plücker, and Stewart, culminating in the complete theory of the subject formulated by Kirchhoff. It was a process of blind groping and much labor, seldom relieved by that *apercu* so necessary for significant research. The report of this search for the secret of the spectrum is inevitably and excusably tedious.

The younger Herschel found that when certain media are used as prisms some of the colors of the spectrum are obliterated completely, showing not a continuous spectrum but a discrete set of colored streaks. By studying sulphur, alcohol, lime salts, and strontium salts heated to incandescense, he noticed that their spectra were different. But his suspicions concerning the bearing of these experiments were not adequately followed up or verified by him. He observed also that when some transparent media are taken in different thicknesses, the color of the transmitted light varies. The quantitative explanation which he offered for this observation is a simple illustration of the *method* of a general formulation of qualitative changes. He showed that the tint and intensity of the rays of light emerging from a medium of a certain thickness, could be expressed as a function of their intensity upon their entering the medium and of the ratios of absorption which a unit thickness exercised upon each ray. The function he obtained showed clearly the dependence of the tint upon the thickness penetrated by the rays. "It is only from a knowledge of the relative values of the absorption ratios in the various parts of the spectrum, that we can say *a priori* whether the tint of thick glass will retain any similarity to that of a thin one."[26]

The aim of Brewster in studying the lines of the solar spectrum was the discovery of a general principle of chemical analysis, by means of which simple as well as compound bodies could be identified through their action on parts of the spectrum. He found, for example, that sulphur vapor absorbed the *violet end* of the spectrum, and iodine vapor the *middle portion.* But it was his experiments on nitrous oxide gas which amazed him completely. For when he examined by means of a rock salt prism the light of a lamp which had passed through a layer of this *gas,* he saw the spectrum crossed with hundreds of dark lines; but when the gas was *liquified* and the lamp light allowed to pass through it, none of the dark lines were shown.[27] Brewster was here on the track of something very important, namely, the absorptive action of gases upon light. Yet the significance of what he observed still escaped him, and he thought he had merely discovered more convenient circumstances for measuring the wave lengths of the dark lines. Nevertheless, by 1836 he could say that he was able to identify many salts through the character of the spectrum they produced. And he could affirm with some assurance, on the basis of maps of the solar spectrum made both with and without the interposition of layers of nitrous acid gas, that this gas must exist in the atmosphere of the earth.[28] By 1860 he had sufficient confidence in his method to decide between three possible interpretations of the Fraunhofer lines: they indicate a deficiency in the sun's rays themselves; they indicate absorption by the sun's atmosphere; they indicate absorption by the earth's atmosphere. Here the logic of scientific method comes into evidence.

[26] *Edinburgh Trans.* Vol. 9, p. 448.
[27] *Edinburgh Trans.* Vol. 12, (1834) p. 522 ff.
[28] *Phil. Magazine,* 3 ser. Vol. 8, p. 384 ff.

The first he thought incapable of positive proof, and to be dismissed in favor of an inference for which evidence could be supplied. If the second interpretation were true, he argued that light passing through a thinner layer of the sun's atmosphere should be less absorbed than by a thicker layer. But the observations of solar eclipses at that time could detect no difference between light from the sun's limb and light from its center. He therefore dismissed the second theory. If the third were true, gases which were known to be present in the earth's atmosphere should be capable of being identified as producing ascertainable cases of absorption. Consequently, his experiments on the absorptive action of nitrous acid gas, which is found in the earth's atmosphere, led him to adopt, at least tentatively, the third alternative.[29]

A somewhat different approach to the problem of spectra was made by Plücker. He found that electricity conducted through gases under low pressure gave a spectrum characteristic of the gas, and that under these circumstances the spectrum of the electrode did not appear.[30] Reasoning upon the basis of a molecular theory of gases, for which evidence could be obtained on other grounds, he inferred that the spectrum of gases is an indication of the nature of the molecules of the gas involved, an opinion which was shared by Angstrom.[31] The suspicion that the analysis of light might bear upon the nature of submicroscopic entities is not of recent date. It was the beginning of a far flung theory which in recent times has come to embrace mechanical, electrical, thermal and optical phenomena.

But a well nigh complete formulation of the principles of spectroscopy was made by Balfour Stewart as a consequence of a series of experiments for the explanation of which he invoked Prevost's theory of heat exchanges. This theory consisted of three hypotheses: If any enclosure be kept at a uniform temperature, any body placed inside of it will finally reach that temperature; All bodies constantly give out radiant heat, at a rate depending upon their own substance and temperature, but independent of the substance and temperature of surrounding bodies; A body at a uniform temperature receives as much heat as it radiates. He showed that all the known facts about radiant heat could be deduced from this theory, and that if light and heat are regarded as involving a similar mechanism of production and transmission, the relevant optical facts also find their place in this system.[32] Stewart compared, first the quantity of the heat (as measured with a thermopile) radiated from polished plates with the quantity radiated from lamp black at the same temperature; secondly, the quantities of heat radiated from polished plates at the same temperature but different thickness; thirdly, the quality of the heat (as measured by the capacity to penetrate screens) radiated from polished surfaces with the quality radiated from lamp black; and finally the quality of the radiation from plates of the same substance but different thickness. He concluded that the emission from any thin slice equals the absorption, and that the heat radiated is the same, qualitatively and quantitatively, as the heat absorbed. These experimental conclusions were also consequences of the Prevost theory.[33] Stewart could legitimately feel

[29] *Phil. Trans. Vol.* 150, p. 158 ff.
[30] *Poggendorf Ann.* Vol. 103, 104.
[31] *Phil. Mag.* 4 ser. Vol. 9, p. 340.
[32] *Proceedings of the Roy. Soc.* Vol. 10 p. 390.
[33] "On Radiant Heat," in *Edinburgh Trans.* Vol. *22* (1861) esp. p. 14.

that hereafter the burden of proof should rest upon those who *refused* to identify the light and heat producing processes.

Meanwhile, the development of photography had made possible, beginning with Becquerel's work in 1842, a more careful survey of the solar spectrum, from the ultra-violet to the infra red. But that the exact significative nature of the spectrum lines was not fully understood is evident from Foucalt's work, who could observe without understanding what he saw, that the electric arc spectrum showed the same lines as *bright* which Fraunhofer had seen as dark in the solar spectrum, or that the bright lines of the arc spectrum appear black when light from glowing coals is examined after passing through the arc.[34]

With the work of Kirchhoff and Bunsen, however, the long series of gropings and lucky guesses was transformed into a full and coherent account of what spectroscopy denoted. In the first place, by using an improved arrangement of telescopes and prisms and by employing a Bunsen gas flame instead of an alcohol lamp, the characteristic spectra of the different metals were studied by them with greater ease, since the spectra of the heating flame were eliminated; and their chemical properties identified. Then it was shown that the characteristic line of sodium coincided with the *D* line of the sun, by allowing a sodium flame to illuminate the slit of the spectroscope already illuminated by the sun, but so that the sun light did not pass through the flame: *The dark line D was actually seen to be transformed into a bright one.* Again, in order to discover how far the intensity of the solar spectrum could be increased without the sodium line disappearing, Kirchhoff allowed the sun rays to pass through a sodium flame, and "saw to my amazement that the dark lines *D* were intensified to an extraordinary degree."[35] The secret of the spectrum was finally revealed, and Kirchhoff explained what he saw by assuming that every gas weakens by absorption those and only those rays passing through it which are like the ones it is capable of sending out when incandescent. Consequently, he reasoned that the spectrum of every gas must be "reversed" if the rays of some bright, hotter source, which forms a continuous spectrum, pass through it.

But Kirchhoff's great service was to show rigorously that this assumption concerning the emission and absorption of light was itself a *consequence* from the more general principles of the kinetic theory of energy exchanges. He showed in a rigorous manner that if it is assumed that bodies radiate only those rays which they can absorb, and if perfectly reflecting mirrors are possible; then for rays of the same wave length the ratio of the emissive and absorptive powers is the same for all bodies at the same temperature, and equal to the emissive power of a black body at that temperature.[36] In other words, the ratio of e to a is a function of λ (wave length) and T (temperature); when the function is not zero, bodies emit light of a characteristic color, and the greater a is, the more light is radiated. The well known fact that opaque bodies become incandescent at about the same temperature, that transparent bodies require a higher temperature, and gases a still higher one for incandescence, finds its formulation in this theorem. Finally, the reversal of lines in a continuous

[34] Winkelmann's *Handbuch, loc. cit.*

[35] *Abhandlungen Berliner Akad. d. Wiss.* 1861, p. 74.

[36] *Monatsberichte, Berl, Akad. d. Wiss.* 1859, p. 758 ff.

spectrum when the light from an incandescent solid passed through a cooler gas, is also a consequence of this general principle.

By means of this principle and its corrolaries, Kirchhoff and Bunson determined the spectra of a great many known elements, and discovered two new ones. With the recognition of the meaning of the spectrum came a need for greater accuracy in its study. And indeed spectroscopy became an altogether reliable means of making measurements, when Angstrom a few years later made a very precise determination of the wave lengths in the solar spectrum. But Rowland's subsequent introduction of the concave grating made previous maps and wave length evaluations useless; his wave length measurements were accurate to within one hundredth of an angstrom. Finally, the use of photography and quartz prisms also extended the range and precision of spectrum studies. It goes without saying, that the measurement of wave length is always surrogative: what is involved is the precise determination of some quantity which is an exact *function* of λ and some other dimension, and the reduction of this dimension to a function of some recognized standard of length.[37]

Thus by the time of Kirchhoff, the spectrum, originally a toy, an event enjoyed as an individualized existence, had gradually become a valuable and reliable *sign* for certain recurrent processes in nature. The individual bright lines were associated with some periodic process finally identified as electromagnetic; *combinations* of these lines were taken as signs of chemical elements; the dark lines became the common ground on which thermo-dynamic and optical studies joined forces; and, as we shall see, the variations in the heating effects of the different parts of the spectrum led on to a novel interpretation of submicroscopic behavior.

(d) *The quantum introduced.* We enter upon the last lap in the progress of spectroscopy toward the quantum concept.

The high precision attained by Rowland in the determination of wave lengths, was followed by an equally accurate measurement of "energy distribution" in the spectrum, achieved by the use of Langley's bolometer or actinic balance. Langley observed that if, instead of measuring directly the energy of the rays which fall upon a thermopile, he were to use this feeble energy as a modulator of some greater force, he could obtain an instrument more sensitive to temperature differences than hitherto available. He therefore passed two wires of equal length and cross section from a battery to a differential galvanometer, so that one current had a tendency to move the needle to the right, and the other equal current to move it to the left; consequently, the needle remained at zero. But when rays from some source having a higher temperature than the wires were let fall on one of them, this wire became heated and its resistance increased in nearly the ratio of the increase in temperature; consequently the needle was deflected by a force which was proportional, in theory at least, to the energy transmitted by the warming rays, and to other constants involved in the battery, galvanometer, and circuit. The instrument which Langley was able to construct on the basis of these simple relations, was sensitive to variations of heat effects in the ultra-violet portion of the spectrum: rays which warm the strips of metal that form the exposed part of the circuit by as little as one millionth of a centigrade degree, produce a measurable deflection of the

[37]Bell, "On Absolute Wave Length of Light," *Amer. Jr. of Science*, 3. Ser. Vol. 35.

galvanometer needle.[38] Thus another instrument, another complication, was added to the number of mediating agencies which stand between the familiar occurrence of colors and the theory which formulates the conditions and correlates of that occurrence. It must also be noted that the evaluation of the heating effects of the different rays in terms of their "energies," is justified only by that long series of experiments and their theoretical formulation which culminated in the doctrine of the conservation of energy. Energy became not so much a common, substantial thing, but a common denominator for expressing numerically the series of equivalent relations which groups of events exhibited toward some standard event. There is little doubt that the meaning and truth of experiments and propositions in physics can be established and can become intelligible only in terms of a larger body of well attested doctrines.

The older Herschel's investigations on the heating effects in the spectrum now were brought to completion. Paschen, Pringsheim, and others, made a careful series of measurements of the energy distribution in the spectra of incandescent solids as a function of the temperature of the solids. The twofold accuracy which was introduced by Rowland's measurements of wave length and by Langley's bolometer in the calculation of the energy distribution, very rapidly produced a mass of data which called for theoretical innovations as well. The spectroscope was transformed again, from being a hand maiden to chemical analysis, into an instrument for inferring the constitution and motion of submicroscopic entities. One of the most significant studies in this direction was that of Balmer. He found, by examining the four lines of the hydrogen spectrum then known, that a numerical formula could be obtained for expressing the sequence of lines when they are denoted by their wave lengths, a formula which was found later to be capable of predicting the position of hitherto unknown lines.[39] But this aspect of spectroscopic inquiry received its greatest development and explanation in terms of atomic models only recently, at the hands of Bohr and others, and subsequent to Planck's work. The distribution of the energy in the spectrum of black-bodies undergoing temperature radiation, received a theoretical formulation much sooner by Wien and Planck.

A black body is defined as one which absorbs radiation of all wave lengths incident upon it without any surface reflection. Like many other concepts in physics, such as "frictionless motion," or "isolated systems," a black body is a regulative ideal which finds its most perfect but only approximate realization in the radiation emitted from the hole of a heated hollow sphere. The temperature of the sphere can be measured, and the radiation from it dispersed by means of a fluorspar prism into a spectrum. With a bolometer the differences in the temperature of the various parts of the spectrum and therefore the differences in the energy transmitted by the various rays can be determined. If the energy transmitted by rays having wave lengths between λ and $\lambda + d\lambda$ (where $d\lambda$ is a small range of wave lengths) is denoted by $E\lambda\, d\lambda$, the numerical values of $E\lambda$ can be plotted against those of λ, and the manner of the variation of the energy with the wave length accurately determined. Lummer and Pringsheim thus obtained characteristic curves for different temperatures of the black body,

[38] *Proceed. of Amer. Acad.*, Vol. 16, p. 343 ff.

[39] "Notiz ueber d. Spektrallinien d. Wasserstoffes," *Ann. d. Physik*, Vol. 25. (1885).

and found that for a given temperature $E\lambda$ has a maximum at a definite wave length.

From the work of Kirchhoff on emission of radiation, and of Boltzmann and Wien on the thermodynamics of radiation, it was possible to deduce a functional dependence of the numerical value of $E\lambda$ upon the value of wave length for which evaluation is being made, and of the temperature of the radiating body. $E\lambda$ was found to be proportional to $\lambda^{-5} f(\lambda T)$, where f is a function independent of the special nature of the substance emitting black body radiation. The mathematical consequent of this formula, (namely, that the value of the energy maximum is found associated with a wave length which is displaced to shorter wave lengths with rise of temperature,) was found to be in agreement with the experimental determination of the curves.

But if the principles of thermodynamics applied to black body radiation were sufficient to deduce a general dependence of the values of $E\lambda$ upon λ and T, they were insufficient to determine the special nature of the function f. Additional hypotheses had to be introduced concerning the behavior, as formulated by the classical principles of mechanics and electro-dynamics, of atomic and sub-atomic entities identified as the source of radiation. By this means, two formulae for the function f were developed: one by Wien, accurate for the shorter wave lengths; and one by Rayleigh, good only for the longer ones. It was Planck's great good fortune to be successful in two things; in interpolating a third formula valid for the entire spectrum, and which reduced to the equations of Wien and Rayleigh for appropriate wave lengths; and in giving a "physical" interpretation to this formula on the basis of the behavior of sub-atomic vibrations producing radiation of energy.

It was shown conclusively somewhat later, that if the behavior of the electronic vibrators was assumed to be in conformity with Newtonian and Maxwellian principles which involved a *continuous* emission of energy by accelerated particles, the equations obtained for the energy distribution in the black body spectrum were in every case incompatible with the experimental determination of that distribution. Upon the basis of Boltzmann's interpretation of entropy as indicating the probability of the occurrence of any thermodynamical state, Planck was able to show, after hesitating at first to use that interpretation, that his interpolated formula was a consequence from the assumption that the vibrating electrons radiate and absorb energy only in integral multiples of a certain unitary quantum of energy. The numerical value of this constant may be obtained from the curves indicating the energy distribution in the spectrum of a black body. The assumption that the value of energy transmitted by each ray were multiples of a certain constant, was not deducible from any principles hitherto employed in physics. Its justification could lie only in this: it "saved the phenomena" observed. The assumption was gradually seen to warrant confidence, when the constant, which was an *ad hoc* postulate for black body radiation, turned up in investigations as diverse as those on photo-electricity, specific heats of solids, the series of line spectra of the chemical elements, and X-rays. The quantum has become a means of giving a unified formulation of otherwise much-disconnected events, and a heuristic principle for the exploration of large portions of the physical world.[40]

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[40]Planck, *Physikalische Rundblicke*, p. 148 ff; Birstwistle, *Quantum Theory of the Atom*, Chap. I; Richtmyer, *Introduction to Modern Physics*, Chap. VII.

CHAPTER IV

NATURE AND CONVENTION

The preceding chapters have shown that the highly abstract formulations of physics are a symbolical representation of relations between the qualitative manifolds of common-day experience. In the present chapter, some of the issues must be considered which are raised by the existence of alternative or of changing formulations that express the interconnections of events similarly qualified. The history of science as well as the contemporary state of physical science indicates how unstable and changing are the systems of principles which interpret these interconnections between unstable and changing events. Ptolemy and Copernicus, Newton and Einstein, Bohr and Heisenberg, are some of the pairs of names which testify that the latest theory may not be the last. There is, therefore, something undetermined, and perhaps arbitrary, in preferring one theoretical structure to another. But how is it that what is, ostensibly, *one* subject matter, is capable of *diverse* formulations? What are the grounds, if any, which dictate the use of one rather than another formulation? And how may the *nature of the interconnections* between classes of events which our formulations attempt to express, be distinguished from the *conventions,* the *conceptual devices* with which these formulations work?

It is the object of this chapter to show that some intellectual difficulties may be escaped by a whole-hearted recognition of the following. (1) There is brought to play upon any given subject matter undergoing investigation, principles or habits of analysis which are believed to have some degree of validity and to merit some confidence, in virtue of their successful use in inquiries *prior* to the given one. But the meaning of those principles must be redefined in terms of certain operations within the *present* subject matter. (2) The arbitrariness which is implicit in approaching a subject matter with one rather than another set of guiding principles, is limited in two ways: by the goal or purpose of the inquiry; and by the character of the subject matter. This arbitrariness, therefore, is the expression of an ultimate metaphysical character; namely, there is an irreducible diversity or a plurality of individuals in nature. Knowledge of nature is possible precisely because there are more or less isolable fields for inquiry. (3) The discovery of constant or invariant relatives between events does not signify that the inconstant has been banished from nature, unless this proposition is understood as a *definition* of "nature." Moreover, while the formulation of relations between events so that it may express the relations between every set events of a certain class, does not state exhaustively *all* the relations which any one event may possess; nevertheless that formulation must be the expression of *some* of the relations which any set of events in that class can be *observed* to possess. Invariants and variables are correlatives; they are distinctions within a subject matter and not separable components each intelligible without the other. A philosophy which renounces every claim to dictate the characters nature should possess and which pretends to nothing more than the

description of discovered traits, must renounce as well habits of analysis which convert distinctions that are the achievement of an inquiry into hypostatic entities that are made to condition the existence of the subject matter of inquiry.

I

The attempts to discover the intellectual tools of one's method have been exercises in proof by *reductio ad absurdum* when the study of the physiological conditions of knowledge has been substituted for a logical analysis of the actual procedure of science. The history of epistemological theories, which, in the last Century and this, have culminated in a widely spread agnosticism as to the reality of an external world, is a record of fallacies of this type. Under the influence of an atomistic, sensationalistic psychology, the mind has been pictured as a passive recipient of discrete sense impressions; and the conclusion has been drawn that only these impressions upon the mind are known, while both the character and the existences of an "external" cause of these impressions are shrouded in an irremovable doubt. For those not content with a solipsism, it has become a dogma in consequence, that "knowledge" is never of things external to our bodies, that it is of certain *associated* qualities occurring in the brain; but the steps of the inference to the external world are exactly reverse the steps of the physical transmission which brought the stimulus to the brain.[1] The difficulties in such a view come to a head in Eddington's treatment of change of entropy as symbolic of the irreversibility of time.[2] Color is for him a purely mental quality, not an ingredient of the real world of electro-magnetic waves. Nevertheless, he insists that the dynamic character of time, of "becoming," is a genuine quality of nature, even though the physical conditions for the recognition of *this* quality seem to be no different from those required for the recognition of any other. To save "becoming" for nature, he must suppose, consistently enough, that in this case the nerve mechanism does not intervene: the dynamical character of time (grown suspiciously substantial for a relativist, when he talks of the "moving on" of time, p. 97) is observed not through the sense organs but by a private door of the mind. "Mind," indeed, turns out to be something with physical properties dwelling in the body.[3] There is, therefore, a double symbolism in Eddington's theory. Color is something that occurs in the mind, although it is symbolic or equivalent in consciousness to the electro-magnetic waves which are its cause. These "waves" have a structure which is known, although they are, too, symbolic of qualities not "known" and unknowable. If in the case of "becoming" this dualism vanishes, it is because "I grasp the notion of becoming because I myself become. It is the innermost Ego of all which *is* and *becomes*."

It is clear, however, that a *knowledge* of neural paths, brain states, and other physical corrolates of knowledge, cannot be used to invalidate the claim that we do possess knowledge of a world wherein neural paths and brains are distinguishable features having an ascertainable behavior. Moreover, the attempt to break through the scepticism which follows from locating whatever we "directly know" in our head, leads only to further questionable and even unin-

[1] Eddington, *Nature of the Physical World*, pp. 270, 278.
[2] Loc. cit. p. 88 ff.
[3] Loc. cit. p. 311.

telligible suppositions. For the knowledge of a world outside our head is declared possible only on the assumption of an isomorphism in the relational patterns of the percepts in our head and of their causes outside of it.[4] But this isomorphism can be verified only by the impossible feat of comparing the relations between the effects with the relations between their unapproachable causes.

The quest for a doctrine which is to make "valid" once for all the inferential procedure of science, when conducted on the basis of a scepticism as to the possibility of knowledge, is self-contradictory and self-defeating. In the first place, it overlooks the fact that even were some such validating principle discovered, the *decision* or *judgment* that it is a validating principle must, on the basis of the dialetic, raise the same sceptical doubts. But secondly, if an appeal is made to self evident, immediate knowledge, a wide entrance is left for every variety of dogmatism and mystic insight. A sharp separation is made, for example, between "hard," self-evident, self-sufficient sense data, possessing an innate cognitive status, incapable of leading on to error and credited as highly reliable messengers from an external world; and a relational complex, presumed to be known independently of every operation conducted in the perceived world, and yet strangely applicable to it. How science can ever be wrong, or how it can correct and modify its procedure, are not easily understood upon such premises.[5]

Wholesale scepticism as a method of discovering truth, as modern philosophy since Descartes has discovered, is therefore hopelessly vain. And indeed, when definite cases of acquisition of knowledge are taken as points of departure for an epistemology, the distinctions usually made in the quest for a general validating principle of knowledge, are soon seen to be artificial and irrelevant to the demands of the actual, inquiring process.

(1) In the first place, whenever an inquiry is going on, there is taken for granted a large body of principles which in *that* inquiry remain unquestioned, even though they are not intrinsically unquestionable. If they remain unquestioned, it is because they are the only *available* instruments which serve to control a whole series of activities.

The discussion of measurement has made evident the need for initial agreements upon the sufficient conditions for a quantitative procedure in terms of the *explicit* behavior of the subject matter studied. Nevertheless, while the discussion has shown also the possibility of less exclusive types of measuring operations, *some* agreement upon the formal requirements for measurement are necessary to give direction to any actual measuring process. Moreover, the study of the

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[4]Cf. Russell, *Analysis of Matter*, pp. 197 ff, p. 227 ff. The discussion of the "similarity of relations" in Russell's *Introduction to Mathem. Philosophy*, p. 55 is the basis for most of his later philosophic reconstructions.

[5]It is the dogma that what is casually most proximate to the brain is most certainly known, which is the root of much recent epistemology. "In general, the nearer the starting point of the physical process which consists in the propogation of light rays is to the brain," writes Joad, "the less likely are we to be mistaken in our inference as to the nature of its origin; but directly we go beyond the bare statement 'The optic nerve is being stimulated in such and such a way' and conclude 'I am seeing an object of such and such a character,' we are drawing an inference and are, therefore, liable to fall into error." *Aristotelian Soc., Supplementary Volume* 9, p. 132. But how, except by inference, is the *existence* of the optic nerve established? And how, except by inference, is it known that *it* is being stimulated? It is only by assuming the physical world which we do perceive by sight and touch that Russell is able to divest himself of the belief "that the physical world is the world we perceive by sight and touch." *Philosophy*, p. 142. Poets are happy if they can get their heads into the sky, Chesterton somewhere remarks, but philosophers insist on getting the sky into their heads; no wonder they get a headache!

development of the quantum concept has indicated the presence of principles which guide the analysis, the formulation, and the control of a subject-matter on any one occasion, even though those principles are never *completely* verifiable or refutable on that one occasion. The use of thermodynamical concepts in the study of optical phenomena, or the employment of spatial changes as the common denominator for the correlation of other kinds of alterations, are two examples of such guiding principles. The study of any theory of physics, as well as the observation that physics has a history, demonstrates the fact that the theoretical structure of any science is not an immediately apprehended aspect of the world revealed in sense perception. For the theory is an expression of the *orders* of relations of coexistence, succession, and mutual dependence of events, apprehended in their qualitative immediacy. Consequently, while every instance of the application of a theory is a test of the theory, the theory is formulated for *possible* as well as actual cases of its application, and is not, therefore, completely verified in the latter. The principles of interpretation which elicit the significative value of any *one* occasion, are not thereby demonstrated to be the only principles which could possibly be employed.

One of the early methods of determining the value of the atomnic electric charge assumed, among other things, a relation known as Stoke's Law connecting the velocity and size of a falling liquid drop. This law was shown *subsequently* to be invalid for some of the circumstances of the experiment.[6] But it is almost a truism to point out that the modifications which were introduced to correct the law for these circumstances, took for granted *other* relations between events which were unquestioned in the new context where *that* law was subjected to scrutiny. The process of getting knowledge is a discursive affair, and *all* our knowledge can be justified only discursively. The attempts to derive knowledge from sensations, or the converse folly of trying to foist once for all some *a priori* relational structure upon a manifold of qualities, result from the failure to note that no inquiry is significant unless framed in a wider realm of accredited knowledge. Cosmologies which seek to speak of the totality of the universe, with the exclusion of none of its parts, have a propensity to become unintelligible. The relations which are assumed to hold between events and taken for granted at one stage of investigation, are not above revision under the stress of new information; but the revision can be performed only if other uniformities are used with some confidence, in view of the successful habits of procedure which they represent. Any *one* proposition of physics may be questioned, but only in terms of others that are not. The road to truth is not a marble stair where security of position is guaranteed by a *single* foundation, miraculously unshakeable. It is a treacherous, unblazed path where security of any *one* step is never guaranteed: the assurance of safety that is obtainable, is based upon the interplay of an indefinite number of factors, each often independently founded, not linearly dependent upon each other, and individually perhaps incapable of bearing much weight.

Some well explored portions of nature may be reapproached from the point of view of an achieved formulation of selected relations between events. In a relatively complete science, such as mechanics, it is possible to set up a formal list of definitions and axioms, and to elaborate these *more geometrico* without

[6]Millikan, *The Electron* 2nd Ed.

further appeal to experiment. It must not be overlooked, however, that apart from a reference to existent physical objects and operations, such an axiomatic is a branch of pure mathematics, a play with symbols with no more meaning than a game of chess. It becomes mathematical *physics* by virtue of an "interpretation" which assigns empirical objects and empirical uniformities between them to the symbols and symbolic operations of the formal scheme. Consequently, unless and until the adequacy of the theory for any given case is demonstrated by a successful application, its status remains hypothetical and its adequacy for possible events is uncertain. From the point of view of the abstractly studied deductive system, the assignment of certain objects and operations rather than others to the symbols of the formulation may seem arbitrary; while from the point of view of events enjoyed in their immediacy, and thus divorced in intent from any relational scheme, the fact that events do "satisfy" any scheme may seem a mystery. Nevertheless, the connection between empirical objects and symbols of a physical theory rests upon a double fact. In so far as we *think* of objects, it is not in their brute immediacy that they are objects of knowledge, but rather in the way they are mutually interconnected. And secondly, in constructing physical theories, it is the observable, recurrent relations that objects of perception bear to one another which become *stated*. The symbolism of the statement is more easily manipulated than the referents of the symbolism. Certain special empirical relations, when once formulated, are then elaborated and studied in their complex interrelations, so that the formulations of *other* empirical relations may be expressed in terms analogous to the formulation of the first set.[7]

(2) The analysis of processes of inquiry drives one to recognize, in the second place, that the existence of principles and habits which have emerged from prior reflective situations is not therefore a sufficient warrant for their continued success in new contexts. Ideas and systems of ideas taken over from one domain may be fruitful or they may be barren in another. The ideas must be redefined in terms of the operations which the new subject matter permits.

Many years ago, Peirce made clear that our ideas are to mean all the experimentally verifiable consequences which follow our acting upon them. Since Peirce was bred in the laboratory and since his philosophy was largely a crystallization of scientific method, it is not surprising that critically conscious scientists, developing their analyses independently of him, have voiced a full-throated endorsement of many of his positions.[8] Thus it has been shown by N. R. Campbell in great detail, that words in common usage as names for objects, depend for their significance upon verifiable uniform associations between certain qualities, and that unless we are assured of the existence of such associations or laws within the domain where these terms are used, we court error and talk nonsense.[9] The extension of terms to situations not hitherto explored involves the *experimental* determination of the existence of corresponding laws. The use of prin-

[7]Cf. Campbell, *Physics*, p. 129; and Gonseth, *Les Fondements des Mathematiques*, Chap. 7.

[8]Cf. Eddington, *Mathematical Theory of Relativity*, p. 3 ff; Born, *Einstein's Theory of Relativity*, p. 61, where absolute space is denied physical reality because nothing ascertainable by measurement corresponds to it.

[9]*Physics*, Chap. 2.

ciples of extrapolation, like the use of other principles of continuity, has its warrant in the concurrence of prediction and observation.[10]

It has been shown in a previous chapter how vagueness of meaning in denotation can be removed by introducing measure-numbers accompanied by an adequate specification of the operations involved, for each number assigned. Qualities, like odors, which do not permit of precise measurement by the methods finally adopted, cannot play a leading role in physical inquiry. The qualities which are more readily manipulable are replaced, however, *for the purposes of the inquiry,* by the methods of their numerical evaluations. The methods of fundamental and surrogative measurement have shown how this can be done. It is an important fact about nature that qualities which cannot be easily and directly distinguished, are uniformly associated with other qualities which are easily handled and capable of sensitive discrimination. It makes science as a project for obtaining knowledge and control possible, since it permits attention to be directed to the *order* of the eventuation of qualities instead of being wholly absorbed in the *eventuations* themselves. But this fact should not be converted into a denial of irreducible qualitative differences, and cannot be used, for any known good reason, to impoverish the qualitative wealth of nature by reducing it to three or four measurable characters. Nevertheless, it is adequate for the purpose of an inquiry to identify the terms with the methods we use for determining them, just as it may be adequate for some purposes to identify telephone subscribers with their telephone numbers. With Bridgman we may say that "the concept is synonymous with the corresponding set of operations."[11]

We turn now to some objections against an operational definition of concepts.

(a) When an idea is operationally defined, not *all* of the conditions of the experiment are or can be stated explicitly, and no *a priori* reason can be given for excluding the particular time, locality, or experimenter from the list of relevant factors. It is, however, a condition for all significant discourse, that as a matter of fact the entire state of the universe need not be taken into account in performing an experiment, and that several experiments may be judged to be the "same" in spite of certain differences important perhaps in other connections. Consequently, the objection, that operational definitions of ideas do not *define,* is well based *if* such definitions were to overlook the rôle of the operation in a wider field of inquiry which determines its adequacy. But there is nothing in the insistence upon exhibiting the operations meant by an idea, which excludes such a reference to the conditions of the problem that defines the value of the operations selected. It is not a mechanical activity that makes an operation synonymous with an idea; it is an activity directed towards the production of further intended consequences. Moreover, the operations which are selected to define an idea are capable of an indefinite number of repetitions if the idea is fertile in serving as a guide and instrument upon many occasions; consequently, it is a series or set of operations which is the *definiens.*

(b) If the concept is synonymous with the defining operations, a second objections urges, the "weight" we determine with a beam balance does not mean what the "weight" determined with a spring balance means, since by definition

[10]The dangers lurking in unverified or unverifiable extrapolations are excellently exhibited by Poincaré, *Dernieres Pensées,* Chap. 1.

[11]*Logic of the Modern Physics,* p. 5.

the operations involved are different. "Length" fixed by the optical properties of some instrument, cannot be the "length" fixed by tactual congruence; and the "charge" determined of an electron obtained with the "balanced drop" method cannot be the "charge" determined from the study of Brownian movements. Why then, the objection goes on, do we use the same name for the number obtained in either way, unless we believe that we are measuring the same "thing," in spite of the differences in operations?

A threefold reply may be made. In the first place, the magnitudes which are obtained by means of a set of operations (irrespective of whether they are obtained by fundamental or surrogative measurement), represent, in most cases, qualitative situations not exhaustively stated by the relational structure in which they therewith take a part. The colors, defined by the operations of determining their wave length, or the whales, classified as mammals on the basis of one of their activities, have a career in existence and have relations with other existences which may be characterized in other ways than those employed in science. It is only for the purposes of a specified inquiry that the identification of a concept with the *actual* operations used can be justified. In any case, however, if concepts are to be significant for guiding action, whether they are identified with actual or possible operations, it is because the observable consequences of using one rather than another concept indicates success or failure in understanding or controlling nature.

In the second place, fundamental measurement, in terms of which as we have seen all other measurables are fixed, require not only certain assymetric, transitive relations between qualities, but require also a specific operation of addition. Consequently, methods of measurement which differ in many marked ways, but which still preserve a qualitative homogeneity in the process of addition, can be regarded as identifying concepts not identically defined operationally. In whatever way the fundamental measurement of illumination may be carried on, there will be no great alteration in the manner in which conjoint activity of sources is produced; and as long as we are measuring lengths fundamentally, many differences in the methods employed will be of little importance, as long as the process of spatial juxtaposition is preserved in their addition.[12]

Finally, different methods of surrogative measurement can justifiably be regarded as measuring the same property, if identical numerical values are obtained by the different methods when they are all applicable in some one domain. The "length" of a road may be obtained by the use of rods and chains, or by triangulations and trigonometric methods, or by the use of optical methods. If identical numerical values are obtained with all the methods, all the methods can be judged to be measuring the same length. The obvious advantage in having several such methods is that one of them may be applicable in a domain where the others are impracticable or meaningless. For example, the thickness of gold leaves can not be computed directly by means of a ruler; but by weighing a number of such leaves heaving a known area and a known density, Faraday was able to compute the average thickness of each leaf.[13] The concept "length" must in some cases, therefore, be identified not with any *one* of the above stated

[12]Campbell, *Measurement and Calculation*, pp. 8 ff; Weyl, *Philosophie d. Mathematik und Naturwissenchaft*, p. 101 ff.

[13]Jevons, *Principles of Science*, p. 296.

operations, but with the *whole set* of them. Nevertheless, there is a real danger that two methods which give identical results in one context may give incongruous reports in others. The unchecked use of extrapolations is always a source of confusion.

The ideal of science to make the explicit formulation of its established information exclude all reference to what is local or less than cosmic, must therefore be adopted with some reservations. For recent work in the sciences themselves has brought into prominence the function of initial agreements, particularly as to measurement, in the formulation of the theoretical structure. Perhaps the most important moral to be drawn from the contemporary revolution in physics, is the fundamental role of denotative definitions in every branch and on every occasion of inquiry.[14] The nature of the particular human investigators, their motives for research, the personal or public value of their discoveries are assuredly irrelevant in a formulation of the order of recurrent relations. Nevertheless, the desire to state the *outcome* of inquiry so that the outcome can be significant independently of its processes and rules is vain. A theory is the answer to definite problems, and theories would be unintelligible and unbearably arbitrary unless viewed in the light of the problems to which they are answers. Were the fabled Martian, innocent of earthly ways, confronted with the formulated doctrines of earth-bound men, his understanding of science and his recognition of its validity would be miraculous indeed, unless the specific physical operations which constitute the meaning of the terms of discourse were also revealed. A view of nature which defines that to be *real* which is constant and invariable for all observers, is as legitimate as any.[15] But unless there is a reference to specific qualitative differences, to ascertainable operations, to human ways of behavior, which make explicit the meaning of concepts, science is a ghostly thing, an unintelligible play with sounds or marks which, lacking a transitive character, cannot be symbolic of anything.

As often happens, very extreme interpretations have been given for this necessity for guiding principles and denotative processes in physical inquiry. To some, the inescapable function of the "human foreground" and their abhorrence of anthropomorphism in science has been the ground for waving aside a piecemeal, perspective approach to nature as an illusion, and for inviting a dumb adoration of her incomprehensible processes. To others, the dependence of the meaning of physical theories upon denoted contexts and special condition of investigation has been a weapon for fortifying a sensationalistic empiricism. The fatal alternative, (disruptive of the continuity between nature and man), between nature and convention has been invoked with as much vigor as in antiquity. "Die Tatsachen sind das 'Natürliche,' die Theorien die 'Kunst,'" writes Petzoldt.[16] And to others still, the selective character of denotative definitions, their reference to apprehend qualitative manifolds, has signified that the diversity which appears to characterize the processes of nature has its source in the creative activity of the human mind.

But the distinction between the natural and the conventional, between what

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[14]Cf. an excerpt from a letter from Einstein, in *Logos* Vol. X, 1921. p. 354.

[15]Planck, *Physikalische Rundblicke*, p. 34. But see the interesting remarks of Thirring upon the anthropomorphic character of physics, in *Handbuch der Physik*, (Geiger & Scheel) Bd. IV, p. 82 ff.

[16]*Relativitäts-Theorie* 2nd Ed. p. 18.

is discovered or demonstrated and what is taken for granted, is hardly a warrant for such conclusions if the distinction is taken as it operates in any actual inquiry. That definitions in the form of a series of physical operations can be made, betrays something about nature as well as about ourselves. It discloses individualized, denumerable processes, capable of indefinite repetition and capable of quantitative analysis. It discloses that we are equipped for turning away from the poignancy of the qualitative immediacy of nature; and that by interacting with those processes we can discover linkages in the interest of perpetuating these qualities. That these linkages in a sense are subteranean and not obviously connected with the surface play of qualities, and that, as a consequence, science is not a photographic reduplication of nature, are lessons slowly learned. We must turn, however, to the examination of the claims that the outstanding uniformities of nature are attributable to the intervention of human agency, to the claims that our knowledge of nature is "conventional."

II

What is the meaning of the distinction between the "natural" and the "conventional"? Does it denote two mutually exclusive categories, which characterize *once for all* every type of existent or relation between existents? Or does it denote a distinction in *analysis* so that the things distinguished may, on different occasions, be characterized in either way? An examination of the various meanings of the word "convention" in current usage, will help to clarify the intent behind the distinction.

(1) The most obvious and primitive meaning of "convention" is the act of coming together for purposes of deliberation, instruction, amusement, or recreation. The character of such meetings is as diverse as the reasons for their existence. In every case, however, conventions in this sense are responses to definite human needs. But what the *consequences* of conventions are, are functions not only of those needs; they are functions of the subject matter as well through which they are satisfied. They may result in the coordination of activities, the destruction of barriers to communication, or the realization of hitherto unemployed potencies. It is not customary to designate conventions, understood in this sense, as arbitrary. But if they are so considered, arbitrariness devolves upon them either because of the special interests and aims which they represent, or because of the particular course of action in which they issue. The existence of a plurality of interests and aims can be characterized in general terms as the indication of an irreducible diversity of existential traits. Actions entered upon with an eye toward fulfilling a prior need are the result of deliberation; although whether the acts selected fulfill the need which prompted their choice, cannot be the outcome of another choice. The role of deliberate choice is the role of any hypothesis, and in so far as conventions exist to solve difficulties, the arbitrariness which may be imputed to them is the arbitrariness implied in the existence of thought.

(2) Conventions as general agreement, contract, customary rule, are an outstanding feature of every civilization. Indeed, they have been identified as the agents of a liberating morality.[17] It is difficult to draw clear lines of demarcation

[17]Cf. Dupréel, "Convention et Raison," *Rev. de Met. et Morale*, 1925.

between activities purely native to the human organism, and those of conventional or social origin. But the existence, as well as the value of such customs or habits, would become objects of wonder if those habits were supposed to be unaffected by the compulsion which the nature of things exercises upon the agreements of men.

The well-known traffic rule which prescribes the right side of the road to vehicles moving in a certain direction may help clarify the interdependence of the customary and the structural or natural. The rule, it may be said, is arbitrary, since there is nothing in nature, human purposes and regulations apart, to prevent vehicles from using the left side of the road. It is impossible not to assent to this comment. But it is of some relevance to point out that the very form of the objection implies a sharp separation of human activity from the rest of nature so that, consequently, any instance of volition is a *deus ex machina,* and involves the usual mystery attending a supernatural event. It is of great importance to note that rules arise to meet specific discontinuities in human affairs and serve as compromises or devices to overcome them. Their efficacy in performing their function is conditioned by character of the discontinuity. Subject matter must be recognized if rules are to be effective.

It may be retorted, nevertheless, that in spite of this, it is still arbitrary if we designate the right side of the road rather than the left in our prescription; there is nothing in roads or automobiles, or desires to avoid congestions and accidents, which points *uniquely* to one side rather than the other. Alternatives are pointed out, but the choice between them is not dictated.

Again, however, assent must be qualified. Whether alternative solutions are possible depends upon how closely a problem is defined. The equation $x^3+x^2+x+1=0$, for example, has *three* roots if we admit negative and imaginary solutions, *one* if our domain is that of real numbers, and *no roots* if we restrict ourselves to positive integers. Either the right or left side of the road may be chosen in the rule, if the *only* requirement which the choice must satisfy is the avoidance of traffic congestions. But if the consequences which follow from the choice of one side of the road be of a nature such that the community would wish to avoid them, the choice between the two sides will no longer be indifferent. It may be, for example, that the use of the right side of the road has as a consequence the gradual paralysis of the left arms of automobilists, or an increase in the rate of road wear. Hence a problem loosely defined may present alternatives in terms of which the initial situation may be *re*defined to eliminate one or more of the suggested alternatives. Alternatives which do not modify the significance of the problem to which they are solutions are only verbally different as far as *that* problem is concerned.

A man dies. What is the cause of his death, is it unique or plural? He may have killed himself, he may have been murdered, he may have met with an accident, he may have had a fatal disease. Clearly the question will not admit a definite answer unless "death" is defined more precisely; and one can argue for a "plurality of causes" only on the basis of an ambiguity in the meaning of the effect. So also to the police it is the murderer who is the cause of the man's death; to the prosecution, it is the revolver; to the physician, it is the severence of an important artery; to the physiologist, it is the starvation of the bodily tissues; to the sociologist, it is the character of the public education. Only

the conditions of the problem which generates the inquiry define and limit the number of significant solutions it may have.

Consequently, the alleged arbitrariness in the existence of rules which require a choice between alternatives is the arbitrariness involved in having definite ends-in-view. That there are such ends-in-view is a fact about that world which includes the human scene. Although that fact is not ubiquitous, the metaphysical status of ends-in-view is the same as the status of any characters whatsoever. One can always raise the question *why* any given quality or uniformity is present rather than another; but since no matter what the quality or uniformity would be, the question could still be put, it has the *form,* but not the *significance,* of a query. It is a safe rule that an inquiry which does not set the standards for its own answers is sheer verbalism.

(3) These commonplace observations have been introduced to show how gradual is the transition from the every-day meaning of "convention" to its meaning as exhibited in scientific practice. Three phases of "conventional" procedure will be considered.

(a) Much confusion has resulted from Poincaré's analysis of the role of hypotheses, in particular of the so-called "unverifiable" hypotheses, in scientific method. His characterization of the choice of geometry as conventional has persuaded many that pronouncements in physics are either tautologies or capricious. But Poincaré's essential point was that a geometry is a hypothesis as to our measuring appliances, a resolution to operate upon and interpret subject matter in a certain manner. To ask whether a geometry is true or false may have no *meaning,* since the symbols and operations of "pure geometry" must be defined in terms of a series of empirical objects. The lengths, angles, congruence, etc, which are defined by means of very carefully prescribed rules are not true or false; they are the empirical *meaning* of the generalized, abstract study of order pursued in pure mathematics.

Consequently, it is always possible to maintain the *truth* of a geometry by the introduction of proper assumptions to account for apparent discrepancies. For if the geometry, now become a branch of physics, were to exhibit contradictions; if, for example, the theorem that the sum of the angles of a triangle is equal to two right angles, which is a dialectical consequence from the axioms, were not verified experimentally when empirically defined entities such as light rays are the equivalents of the "lines" of pure geometry; and if we resolve to maintain this meaning of "lines" in terms of light rays several alternatives present themselves. We may shift the burden for the discrepancy upon faulty calibrations of the standard series of angles; or upon "errors" of measurement if the discrepancy is less than the accuracy which our procedure leads us to expect; or upon some hypothetical forces which deform the linearity of the light rays or the measuring apparatus.

Nevertheless, the truth of the geometry has been won at the price of recognizing new factors, new influences; in the last case supposed, deforming forces had to be introduced. Now whether we separate "geometry" (the study of the spatial relations bodies bear to one another when certain influences are not operative upon the bodies) and "physics," is a matter for the classification of our knowledge. We may, however, redefine our foundamental geometrical concepts, and thereby avoid later the need for recognizing such new influences and

give greater coherence to the formulation of physics. A "straight line" may be so defined that *whatever* be the path of a light ray, that path will be a straight line; nothing, in other words, can deform the linearity of the light rays. This is just the procedure adopted in the general relativity theory. Instead of recognizing special fields of force which introduce *modifications,* and which appear therefore as something extraneous to the geometry initially defined, the fields of force are introduced initially with the geometry. Geometrical concepts are so defined that throughout the domain of their applicability those fields of force shall vanish.[18]

It is, therefore, something less than the whole story to maintain that conditions of symmetry "may be entirely created by conventions as to measurement."[19] The formulation of a symmetry is conventional, but, as in the case of simplicity, if it is not a "property of the real world," it can be maintained only at the expense of subsequent complexity and by temporarily disregarding factors which finally *compel* our respect. It may be possible to construct a theory of human conduct upon the basis of some one single principle, whether it be that of self-preservation, of sex gratification, or of altruism. No instance of human behavior need lack an explanation in terms of the principle of the theory. But every theory will have to mold itself upon human behavior as it is *discovered;* and some such theories may find themselves in a continual state of *ad hoc* postulation in order to account for behavior more simply explicable on a theory which may be *initially* more complex.

(b) The abandonment of a regulative idea which has come to be fixed in custom is often the cause of much dismay, and it is undeniably disconcerting when the new practice is evaluated on the basis of the habit explicitly abandoned. A frequent source for paradox is the use of measurements to establish comprehensive theories which lead, however, to a *correction* of the measurements upon the basis of arguments drawn from the theory *they* have been instrumental in founding.

The subject is too large and the requisite knowledge is lacking for making more than a passing reference here to the problems involved. In a few simple cases, however, it is not difficult to escape from the maze. For example, the chapter on measurement has shown that at some point in physical research measurements must be made that are *fundamental* and not dependent upon any numerical laws for their meaning. Volumes and masses of liquids can be measured by such a process, and subsequently a numerical law can be established between them which expresses the constancy of the liquid density. But if this law is now taken as the basis for making new measurements, it may lead to more accurate, because more uniform and converging, evaluations of masses or volumes than the fundamental methods made possible.

The measurement of time is a condition for the establishment of dynamical laws. Some periodic process is discovered which is defined to beat out "equal times"; some of the periodic processes that have been used are the pulse rate, the passage of a quantity of sand through an opening, the swing of a pendulum,

[18]A detailed account is given in Reichenbach, *Philos. d. Raum-Zeit Lehre,* especially Part one. See also Campbell, "Principles of Dynamics," *Philos. Magazine* (1910) 6, Series, Vol. 19.

[19]Russell, *A.B.C, of Relativity,* p. 184.

the diurnal rotation of the earth with respect to the sun or to the stars. The laws of mechanics were discovered and stated upon the basis of some such time measurement, and in fact upon the use of the earth's rotation as a clock. Nevertheless, upon the basis of the laws of mechanics, and in order to account for irregularities in the motion of the moon, it has been calculated that the earth is retarded from eight to twenty-two seconds per century. There is a logical alternative, however, of continuing to regard the original clock as "accurate" and the laws of mechanics as only approximately true, or of maintaining the truth of these laws and "correcting" the readings of the clock. The choice in favor of the latter procedure is in part conventional, but in a larger measure is dictated by the unfailing prognostic power of the laws in *other* contexts.[20]

Finally, the paradox has been often noted in using instruments, constructed and interpreted ostensibly on the principles of Euclidean geometry, to verify physical theories which operate in terms of a non-Euclidean space. The paradox is at least partly removed when once the heuristic function of a geometry is understood; for a geometry which is an adequate guiding principle within one domain may be abandoned in favor of another geometry which is more adequate for a more inclusive domain. The operations and theorems of the old geometry may, nevertheless, be interpreted on the basis of the new one. Moreover, and with this observation the paradox may vanish completely, the differences between Euclidean and non-Euclidean geometries, when interpreted operationally in small domains, may be insignificant; instruments constructed and operated on Euclidean and on non-Euclidean principles may exhibit no observable differences within certain bounds. Consequently, a Euclidean geometry may be used within small regions without committing a logical error, on the understanding that it is an excellent approximation to the geometry which serves to guide inquiry in the wider domain.

(c) We are thus gradually brought to a realization of the heuristic function of scientific concepts and theories.

It is a common practice to formulate a relation experimentally determined in a certain domain; and then to *define* a similar type of uniformity within a more comprehensive region, so that when conditions of verifiability obtain, the relation can be determined to hold. This can be seen in the history of the doctrine of energy.[21] The definition then functions as a hypothesis with which we survey and correlate events having a more extensive scope than we are capable of confirming experimentally. Such hypotheses take the form of guiding principles which may prove fertile or not in suggesting verifiable relations, but which cannot be overthrown by any crucial experiment. A generalized principle of conservation of energy thus acquires a purely formal validity, and does not specify the number of energy forms which can be obtained. Its importance lies not in summarizing the cases in which the relations between energy variations have been found to preserve constant form; its importance lies in its directive value for guiding search for quantities which will be found to conform to the principle. In the case of mechanical systems, those quantities are functions of the masses,

[20]Cf. the illuminating analysis of this subject by Silberstein, *Theory of Relativity*, 2 Ed. p. 8 ff. Also Tannery, *Science et Philosophie*, Chap. II, especially p. 24 ff.

[21]Cf. Planck, *Princip. d. Erhaltung d. Energie*, 5 ed. pp. 100, 111. Also Campbell, "Physical Doctrine of Energy," in *The New Quarterly*, Vol. 1.

positions, and velocities of the particles; in thermal studies they are quantities of heat; in electrostatic systems they are functions of the value of the charge and the potential; in electrodynamic systems they are functions of the current strength and the potential difference.

It becomes clear, moreover, why, when conservation laws are generalized to apply to qualitatively dissimilar contents, the conservation turns out to be not of some *thing*. For the conservation of a thing is very uncertain experimentally; and there is no clue offered by the principle for its possible discovery in different contexts. The laws express the conservation in terms of the equivalence between sets of events with respect to their relation to some given event. A chemical reaction and the passage of an electric current may be equivalent with respect to certain temperature differences produced.

The manner in which such guiding hypotheses are developed and used demonstrates clearly the shallowness of those philosophies of scientific method which give central importence to crucial experiments. For the distinction between fact and hypothesis is a shifting one, shifting with the context in which it operates and with the stage of inquiry reached. It is the fixation of guiding principles in the form of habits which compels agreement with respect to what is a fact and what is not. But however much such habits reflect non-human connections in nature, there is no assurance, until the last event in the universe has confirmed them, that they may not, some day, need revision.

It follows, therefore, that the inference to the conclusion that the theoretical struction of physics is exclusively a function of the human mind cannot be well founded. "Ultimately," says Eddington, "it is the mind that decides what is lumber—what part of our building will shadow the things of common experience, and which has no such counterpart."[22] It is his failure to indicate in what contexts and selections are made and what problems are satisfied by the solutions offered, which makes his account so very unilluminating. In one sense, everything is decided by mind, since deciding is something that only a mind can do. The force of Eddington's insistence upon the truistic character of physical laws is gone, however, if one remembers that decision and choice can arise and can be effective only when subject matter possesses traits which may be chosen. To say that the prominence of conservation laws is due to the mind's demand for permanence is doubtless important; some things are conserved only because other things are not, and if we study the conserved things we certainly select. But if we were to study the things which are not conserved, as we sometimes do, we would be exercising selection also. Since the mind is *always* selective, no light is thrown upon the metaphysical status of any entities by pointing out that they are *selected*. The relevant question, the only one of value is why or in what context the selection is made.

Moreover, the thesis that the laws of field-physics are only truisms is a bad pun. The argument is that concepts like matter and energy obey these laws only because matter and energy have been so defined as to obey them. But this argument overlooks the processes, the experiments, the hesitant gropings for light, which anteceded the formulation of the principles. And on this argument, every *conditional* equation would be an *identity* and a truism, because its roots

[22] *Nature of the Physical World*, p. 235.

when they are found identically satisfy it upon substitution. The truism is manifest, therefore, only upon a condition; but the condition is just that one which permits us to say that "the sky is blue" without thereby making "sky" and "whatever is blue" identical.

Conventions are, therefore, Janus-faced. For in addition to the unique nature that they possess as definite operations in existence, they are conventions *for* a certain field of inquiry, they look *toward* subject matter; and at the same time they are directed *to* a group of inquirers, defining the point of approach in terms of which the objects of inquiry can be initially fixed. Because conventions are *for* a certain subject matter, structure independent of human volition is implied. Because conventions refer *to* beings having purposes or ends-in-view, they are *a* way of considering subject, *a* set of operations, *a* selection from the rich bosom of nature.

Conventions, like concepts, are not merely psychological, impositions of some order upon an amorphous flux, and whose significance is exhausted by their human reference. It is true that science is, in an obvious sense, a mental product, and that the objects of our knowledge are not found ready made. What is at our disposal at the beginning of an inquiry, is a function both of the subject matter and of the skill and imagination of the investigator. What elements may serve as data for the resolution of the set problem, requires insight and experience to discover. But that the data are *sufficient* for the task is not a matter of choice or agreement; *that* is determined by the subject matter as inference and experiment disclose.

Against Vaihinger and his school, who find in the alleged self-contradictory nature of "fictions" a proof of their impotence to come to grips with reality in spite of their usefulness, it may be remarked that "usefulness" as an explanatory category has been shown to have metaphysical bearings as long ago as Plato of the *Cratylus*.[23] The data of science are defined by certain operations, are modified on further inquiry; and may well be said to be manufactured, and would lose their meaning if separated from the inquiry for which they are designed. But the data are given certain functions only because the subject matter can be manipulated to yield them.

When we fix certain data for points of departure we do not thereby fix the order of all discoverable events. We may *virtually* select some connections between events rather than others, by selecting one point of departure instead of another. But what connections we do so virtually select, only exploration of the subject matter can decide.

The presence of conventional elements does not therefore vitiate the procedure of science at least not on any theory which recognizes that one may have valid knowledge without knowing everything. Knowledge may be called absolute in the sense that unambiguous answers are discoverable to unambiguous problems. But knowledge that is absolute in the sense of having no bearing on *any* specific issue, although it may be the despair of a romantic pessimism, is no provocative to lamentation for those who understand what we do when we know.

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[23]For a modern defense of "abstractions" against some of its critics, see M. R. Cohen, *Journal of Philosophy*, Vol. xx, p. 477 ff. The metaphysical bearings of "usefulness" are brought out in Woodbridge, "Natural Teleology," *Briggs Memorial Volume*.

III

Much has been made, and justly so, of the relational character of scientific ideas. Science is not history. It does not accumulate facts to distil their unique flavor. It concerns itself not with particulars in their concrete existence and individual careers. Its professed and performed task is to show linkages, discover unsuspected correlations between things diversely qualified; its concern is to exhibit the *form* events possess as functions of other events. Consequently, it is never content to leave fields of inquiry disconnected; it aims to find general methods by means of which a unified doctrine can be stated which holds for all occasions of inquiry. Although subject matter is delimited when studied, the goal of science is to push back the limits indefinitely by coordinating diverse fields.

In physics this synthesizing process finds many illustrations. Not only is its theoretical structure transformed from time to time by the "reduction" of an independent domain to a branch of another, e. g., electro-magnetism to a branch of mechanics, mechanics to thermo-dynamics, and conversely. *Within* some one branch of physics a fertile generalization may be introduced, so that the special conditions of observation, the special manner of approach play no explicit role in the formulation of the discovered uniformities. In the study of motions of bodies, for example, the equations of motion can be so stated that the motion of a body is expressed not only when it is referred to the sun; irrespective of the orientation, the velocity, or the acceleration which the frame of reference may have with respect to the observer, the mathematical formulation of that motion preserves a constant form. This remarkable fact has been stated as the principle that the equations of dynamics shall be convariant under every group of continuous transformations of the coordinates.

The notion of an invariant under transformation plays a dominant role in recent theories. But while the notion has become a conscious principle of research in only relatively recent times, and while in its technical form it is quite complex, the notion has been used the first time an idea was communicated. It is a canonization of the fact that a statement can be made which is the adequate expression of the relation classes of events bear to one another, without being an exhaustive expression of *all* the relations any *one* of these events manifest.

Philosophies of flux, from Heraclitus down, have made the belief in an unchanging thing or material substance difficult to hold, and recent science finds little reason to endorse the permanence of any physical object. Even though objects like pyramids or "fixed stars" may display to successive generations of investigators an uniformity of appearance, all that we know of the behavior of matter, all the requirements of our most reliable theories, make it hazardous to affirm a "substance" notion of invariants. Moreover, it is questionable whether the constancy which is noted in the series of appearances of such objects is an immediately apprehended qualitative duration; and the intent of the predication can be communicated only if reference to *ratios* of rates of change is made. The invariance which is discovered in nature is the constancy of types of processes.

The direction designated as "up" in New York is qualitatively different from the direction so designated in China. If we can convey by language what is meant by such a direction, it is because there is a repeatable process, such as

"the erection of a perpendicular to a spherical surface" which expresses the relation of "up" to other recurrent processes. So also a *definite* procedure must be followed in order to light fires, however different are the occasional settings for which fires are lighted. Paper must be gathered, matches rubbed, wood must be added. This procedure may produce the fire with unfailing regularity; it is then an invariant in those occasions in spite of the variations of circumstances which differentiate those occasions. If, moreover, the regula for obtaining fire can be formulated in terms free from emotional connotations associated with human activity, in terms of the relations which classes of objects bear to one another, this humble example of an invariant process may win an important niche in the gallery of science.

Invariants are best studied, however, in their mathematical dress. For the symbolic formulation of processes makes their mutual interrelations more evident. There are surely very few who have not been thrilled by the study of invariant properties in the different branches of mathematics. Two very elementary examples may be mentioned, one from geometry and the other from elementary algebra. Consider four lines, concurrent in a point O, and intersected by another line A in points a_1, a_2, a_3, a_4. Through these four points draw four other lines, concurrent in P, and let them be intersected by a line B, in points b_1, b_2, b_3, b_4. Then it is shown in projective geometry that there is a function of the four points a whose numerical value is equal to the function of the points b which has the same form. This function, the anharmonic ratio of four points, is an invariant under projective transformations; all sets of four points obtained from the points a in the manner indicated will have the same anharmonic ratio.[24]

The second example may be more familiar. The quadratic equation $ax^2+2bx+c=0$ is known to have two roots given by the formula $x=\frac{-b\pm\sqrt{b^2-ac}}{a}$. Let us subject the x to a "linear transformation" such that $x=Ax'+B$. The equation will then be transformed into $aA^2x'^2+2(aAB+bA)x'+(aB^2+abB+c)=0$, which may be written as $a'x'^2+2b'x'+c'=0$ and whose roots are given by a formula of the type already stated. It is well known that the "nature of the roots" of a quadratic equation depends upon the value of the "discriminant" (b^2-ac) which is found under the radical sign. The roots are imaginary, rational, or irrational, according as the discriminant is less than zero, is a perfect square, or is not a perfect square; the roots are equal if the discriminant "vanishes." But a simple calculation shows that $(b'^2-a'c')=A^2(b^2-ac)$; consequently, the "nature of the roots" is an invariant under a linear transformation of the roots.

The development of mathematics in the Nineteenth Century resulted in such a far-flung mathematical empire that no student could hope to master the entire realm. As late as Gauss a mathematician of genius could become an authority in all branches of the subject, but the subsequent prolific growth of the science threatened to compel him to be a specialist confined to a relatively small portion of the realm. Fortunately, there arose the study of invariants and

[24] The anharmonic ratio of the four points a *is* defined as $\frac{a_1a_2}{a_1a_4}:\frac{a_3a_2}{a_3a_4}$, where for the purposes of this illustration a_1a_2 may be taken to indicate the distance between a_1 and a_2.

the groups of transformations with which they are associated.[25] In the hands of Felix Klein the group concept became a device for formulating a program for the correlation of different branches of study. It was discovered that every geometry has a group of transformations which leave the characteristic properties studied in that geometry invariant under the group; and conversely, for every group of transformations there can be found a geometry in which certain properties remain invariant under the group. Thus in ordinary three dimensional geometry, the class of rotations and translations of rigid bodies forms a group; under this group the characteristic relations studied,—congruence of angles, of segments, equality of areas and volumes,—remain invariant. However these relations of congruence and equality may be defined as precisely those which are left invariant by that group of transformations.[26]

Mathematical invariants are therefore a highly sophisticated symbolic expression of certain relations discoverable between events or between relations of events. Invariable properties cannot constitute an exclusive subject matter, since, evidently, they require variable properties with respect to which *they* are fixed. Nevertheless, the discovery and formulation of such invariant properties is the one way there is for eliminating the particular, the isolated, the merely occasional, *not from existence,* but from the pursuit of the understanding and control of what exists. It is the discovery of invariant processes or relations which makes it possible to break through the verbal and conceptual antagonism of theories mutually incompatible but all somehow applicable; for the discovery of such invariants locates the processes in terms of which the distinctions of one theory are convertible into the distinctions of another.

The most powerful mathematical instrument invented so far to eliminate the particularity of the approach to a subject matter from its formulation is the tensor calculus. It has been repeatedly hailed as the means by which we may escape from a provincial physics to a knowledge of the truly universal laws of nature. In its mathematical aspect it is the study of certain differential invariants. By its help any set of coordinates may be translated into any other set so that certain functions of the coordinates shall remain unchanged in whatever coordinate system it is interpreted. The calculus is therefore a kind of universal language which attempts to eliminate what is parochial in any one selection of measure-numbers and which concentrates upon what is common to all. In assigning measure-numbers in physics we make use of operations which should not enter explicitly into the equations of the theory. It is advantageous to avoid connections between coordinates which issue *solely* from the way they are defined and manufactured. The importance of tensors in physics is to attune us to a universality which escapes us so long as the accent of one set of measure-numbers is permitted, so to speak, to drown out the grammar common to all.

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[25]A set of transformations is loosely defined as a "group" if the product of any two of them is also a member of the set. Thus the class of rotations of a sphere around its center forms a group, because any two successive rotations are equivalent to a rotation of the class. But the class of additions upon the set of numbers less than ten does not form a group, since the successive addition of numbers from the set will produce a number not belonging to the set.

[26]See F. Klein, "Vergleichende Betrachtung Uber neuere geometriche Forschungen," *Math. Annalen,* Vol. 43 (1893); also the interesting essay of Rougier, *Philosophie Geometrique de Henri Poincaré.*

"The which is common to all, and which is equally a part and in the whole, does not constitute the essence of any particular thing."[27]

But it is a curious turn of events which one might suppose occurs only in a realm of fancy, did not the history of thought and contemporary discussion give other evidence, that on attaining a comprehensive outlook men are led to deny the validity of a more restricted generalization. It is by a strange irony that a method which attempts to remove a dogmatic *priority* for any one approach to a subject matter, should persuade some that *no one* approach has validity because *all* are equally valid. It is a surprising reading of the outcome of science, but not an unusual one, that concludes by converting laws of physics whose derivation and meaning require the existence of individualized events, into hypostatic entities with a superior claim to exclusive reality than the latter.

The discovery that bodies have behavior patterns cannot, however, be perverted by making those patterns the *conditions* for the existence of bodies, or by imagining those patterns to stand over against the bodies whose behavior is the full meaning of the patterns. The world of theory is not a duplicate of the world of sense, and cannot be *compared* with it. The world of theory *is* the world of sense in its logical interconnection. It is no more real or intrinsically more important than the world of sense; and the belief that it is, imputes to the outcome of an inquiry for causal factors an importance it does not possess outside of that inquiry. Bodies are found in relational contexts, and relational contexts are relations between bodies.

That there is a reference to the *individualized* behavior of bodies even in an invariant formulation of physics, is evident if the derivation of the equations of motion for a given "free body" from the tensor form of the law of gravitation is carefully examined. The *symbols* of that law are constantly compared with other symbols which have an ascertainable empirical meaning.[28] For without such a "matching" of the symbols of the general theory with experimentally well-founded ideas, it throws no light upon what we are to expect in actual cases. If the equations of motion retain their form in all coordinate systems, it is at the price of neglecting a host of observable feature in actual motions. The formal simplicity, which the equations retain for the general formulation, must be supplemented by the reintroduction of parochial considerations, when their meaning for special cases is sought.[29]

A symbolic expression of relations between classes of events requires no specification of any one event, since the general formula is expressly devised to exclude such a specification. But the existence of such individualized events is indispensable if the expression is to be truly *symbolic*. It can lead only to confusion, therefore, to maintain that the external world is the *synthesis* of appearances from *all* points of view. For if the external world is the synthesis, then any one of the "appearances," any one of the particularized space-time frames of reference, is not that world or a part of it. But if the synthesis acquires significance only by virtue of interpretations from such particularized points of view; and if certain characters in such points of view cannot be taken as exem-

[27]Spinoza, *Ethics*, Part II, Prop. 37. The account of the tensor method in Chap. II of Eddington's *Mathem. Theory of Relativity* is a marvel of grace and elegance.

[28]Eddington, *Math. Theory of Relativity*, pp. 82 ff. & 105 ff.

[29]Cf. Silberstein, *Theory of Relativity*, 2. ed. pp. 365-70.

plifications of that synthesis; then a quest for that synthesis is a quest for a hypostatic external world which must forever elude us.

The "colorless" patterns of science are not indeed the "colored" content the patterns of which they are, any more than *knowing* things in their interconnections is identical with *sensing* them. Because knowledge is concerned with the coexistential and sequential order of events, however, there have not been lacking resigned but melancholy sighs that science can tell us nothing of the "true nature" of things, since it is things in their relations that are known.[30] However, the heroic remedy of dissolving things into relations, of displacing "substance" by "function," sometimes proposed to give comfort against such reflections, is not certain to produce the desired relief if we remember that invariant relations without a field of variables are nonsense. The substitution of a selected aspect from the full, existential concatenation of events, for the events from which the selection is made, is confusion as well as insensitivity to the inexhaustible richness of nature.

Eddington has a tale of the discovery of a volume containing the *rules* of chess, a game supposedly forgotten and no longer cultivated, but giving no indication of the shape, size of pieces, or the color of the board. "It is obvious," he remarks, "that no amount of study of the volume will reveal the true nature either of the participants in the game—the chessmen, or the field of the game—the chessboard."[31] Similarly, we are told, no study of experiments can reveal the obsolute nature of the participants—the electrons and point-events. But (1), what does the assumption that things have "true natures" not disclosed by experiment *mean?* And (2), what is the significance of the declaration that the formulations of physics are *symbolic,* yet so lacking in a transitive reference that they do not reveal the "true nature" of the world in which we live?

(1) "True natures" may signify something that is unknowable; or something that is not yet known but knowable; or that it is not an object for *knowledge* at all. If the first is meant, it is difficult to understand why anyone, least of all a physicist, should concern himself with unknowable true natures, or even know that there are any such things. If the second is meant, there is no reason for despair provided we are told what are the criteria by which we may recognize them. But if, thirdly, it is meant that true natures are unique qualities noncommunicable, to be immediately apprehended in some ineffable experience, it is well to remember that it is not the function of *knowledge* to reduplicate its subject matter or to *be* what it knows. One can be in love without knowing what it *means* to be so engaged; and one may perhaps acquire a knowledge of its significance, e.g. by reading Plato, without ever being wounded by the arrows of the god.

(2) It is the insidious assumption, however, that the physiological correlates of knowledge are really its objects, which generates the doctrine that only the "structure" of the "real" world is obtained by science and that the nature of the "intrinsic quality of things" (except in the case of our own ego) is an inscrutable somewhat not known by us.[32] Proponents of such a view must declare

[30]Cf. Lenzen in the *University of California, Publications in Philos.,* Vol. 9. "The intrinsic nature of physical substance is problematic." p. 170.

[31]*Space, Time and Gravitation,* p. 184.

[32]Eddington, *Nature of the Physical World,* p. 254.

that the world of physics is symbolical, not of things in daily experience, but of some inner essence of electrons and point events.

However, in order to *be* symbols, symbols must have reference beyond themselves, even though, at the same time, they have specific characters of their own. Consequently, there seems no reason for calling the world of physics *symbolic,* if we are never permitted to know what the symbols stand for. The moves in a chess game may be symbolic of army maneuvers, and the numerals on a page may be symbolic of counting. If we never know what the moves in chess or the numerals represent, it is not as *symbols* that they studied, since their referents do not determine the course of the inquiry. Either the referents of the world of physics do condition the symbolism, and then we do know what they are, or physics is not symbolic. Unfortunately, "knowledge" is a word with many meanings, as we have just seen. It is easy to argue, therefore, if one assumes the dogma that only the immediate is real, and if one understands by knowledge the possession of the representative function of symbols, that the symbolic character of knowledge makes it incapable of grasping reality. Nevertheless, it seems permissable to retort that knowledge, when it is taken as the inferential function, is symbolic as well as adequate to grasp reality: the psycho-physical carriers of meanings are not "like" the meanings they carry, nor are the meanings "like" things; but the meanings are meanings *of* things and symbolize things in the sense that they denote types of experimentally obtainable reactions between things.

It is an untenable separation of theory from its applications which has led some physicists to formulate what might be called a "phenomenalism of the instrument," and to maintain that physics knows nothing of the world but pointer readings. "The physicist's knowledge *bears* upon the nature of matter but it *is* a knowledge of pointer-readings only." Such a view is based upon a misunderstanding or a gross neglect of the theory of physical measurement. For it argues that since all of our scientific information is summed up in measures, and since measure-*numbers* as such offer no ground for recognizing qualitative distinctions in nature, therefore no such distinctions exist for science. But while it is true that at a certain stage of investigation all that the scientist may use are measure-numbers, and be indifferent as to how they are obtained, surely a philosophy of science cannot stop at that stage, if indeed it is even possible for science to do so. It is only when the locus of those measure-numbers is discovered in certain qualitative continuities, familiar enough in our daily activities, that the predictive power of science becomes intelligible. An extremely shortsighted behaviorism is the basis for the statement that "two tons *is* the reading of the pointer when the elephant is placed on a weighing machine," since an adequate statement of what a pointer reading is involves the admission of all those qualities which had been supposedly excluded.

Scale readings, as has been amply shown, are calibrations which involve, and are taken as the signs of, properties of matter explored independently of many theories subsequently elaborated. Pointer-readings function like all signs; they are discriminated only after much labor, and used for safeguarding inference. To take them as the primary subject matter of study and thus generate problems concerning the reality of the flux of qualities, is on the level of a philosophy which takes cases of perception as cases of knowledge, and then finds the exist-

ence of knowledge and error a problem. The subject matter of physics is not identical with the data with which the physicist works.

The distinction between more inclusive and less inclusive formulations of subject matter must not be converted into a distinction of degrees of "reality," and the introduction of the very term into discussions on physics produces nothing except confusion. Eddington writes: "He who doubts the reality of the four-dimensional world can only be compared to a man who doubts the reality of the penny, and prefers to regard one of its innumerable appearances as the real object."[33] But a problem in science is never solved nor is any issue settled by relegating either events or relations between them to an undefined or arbitrarily selected category of reality. The question at issue in the instance cited by Eddington is not one of reality against appearance; it is the question of locating the problem to which a four dimensional theory or a three dimensional theory are possible answers, and of determining the meaning of either theory upon the basis of operations it employs. Advances in science do not consist in progressively unveiling reality in the sense that layers of appearance are discovered; they consist in making knowledge more secure, more inclusive, and more unified. The success of a new theory does not relegate the old to a realm of appearance, nor does it by an act of miracle transform the truth of the old into a falsehood. A physical theory is always entertained hypothetically, to summarize and coordinate certain special relations already noted, and to serve as a guide in further exploration. In so far as a theory is thus successfully exploited, its truth is never impugned within the field which was its original province. A new theory may be more comprehensive, better articulated, and assert connections of which the one discarded took no cognizance. But on pain of committing the fallacy of affirming the consequent, within the domain in which *both* theories operate successfully, neither can be supposed to disclose reality to the exclusion of the reality disclosed by the other.

The world contains more than science affirms it to contain. Poetry, music, love and friendship, teach us much which neither the laboratory nor the study of the mathematician can discover for us. Science, too, may reveal to us characters of nature which are in excess of the bare minimum characters nature possesses. But if we do ask what pervasive features of nature science does disclose, the answer is that science affirms nature to be both a realm of necessity and a realm of individual events. The world is much more than this, and a survey of it which reports these features to be all there is, is arrogant and blind in its claims. But because such a meager report, or because a less meager science, does not exhaust the wealth which is nature, does not make either partially false.

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[33] *Space, Time and Gravitation*, p. 182.

CHAPTER V

LOGIC, INTUITION, AND CONSISTENCY

The existence of mathematical and logical activity has been ever a visible sign of faith to those who believe in things unseen. Yet agreement upon the subject matter of these disciplines and upon the metaphysical status of that subject matter never has been complete even among their professors. The number of definitions of logic and of mathematics is legion. At one extreme is the view that mathematics, coalesced with logic, explores a platonic heaven of possibility which in no way is contingent upon the discoverable orders in the world wherein we live and die, but which, on the contrary, limits and conditions whatever may be actualized therein. At the other extreme is the interpretation that logic and mathematics, being human activities initiated and maintained for the purpose of making man's existence more secure, reveal no generic characters of that nature which embraces man's existence, and indicate at best only certain propensities of the human organism. Mathematics and logic are then sometimes declared to be products of the intellectual imagination, and hence to have their basis in psychology; or they are regarded as vicarious fulfillments of man's desire for immortality, in so far as mathematics exhibits that which is unique as a recurrent term in a predictable series of terms.

An adequate philosophy of mathematics and logic must, however, do justice both to their "non-existential" character and their deductive fertility when they are pursued dialectically, and to the undoubted success of this dialectic in dealing with existence. In this chapter, the extremes indicated will be avoided by extending the hypothesis on the status of theoretical physics, sustained in the previous chapters, to logic and mathematics as well. Logical and mathematical systems are the most general formulations, and the intellectual elaboration of the formulations, of certain relational characters discoverable in the common-day world. Consequently, the symbolical expressions of logic and mathematics should involve both certain generic, irreducible characters of existence, and the nature of the problems which require the selection, the isolation, and the statement of those characters. It is almost unnecessary to disavow any intent of even sketching here that survey of logic and mathematics which would make this thesis plausible. This chapter merely undertakes to dispose of two issues which stand in the way of accepting that thesis as a *program* for further study. It will be maintained, therefore, that (1) there is an essential continuity in the methods of investigation used in physics and in mathematics, in so far as valid knowledge is *discursively* reached and is not guaranteed *once for all* by anything that is immediate; and (2) mathematical and logical characters, although they cannot be regarded as exclusive metaphysical characters, must nevertheless be admitted to be based upon certain general characters of existence, if *any* generic characters of existence are admitted as objects of knowledge.

I

Intuition as a special intellectual capacity is a well worn word, and in philosophic and scientific doctrines has meant many things. The search for an indubitable foundation for many doctrines has often terminated in an immediate grasp of simples of various kinds. But the array of data alleged to be intuitively known is impressive; among them have been sensations, inner states of the soul, essences, God, logical principles, the past, moral goods, the principles of mechanics, and the freedom of the will. Consequently, there is probably no proponent of intuitive knowledge, whether this be understood as a sensory apprehension or as an intellectual affirmation, who will indorse *all* these items as possessing a self-evident character.

Indeed, the doctrine of indubitably given truths, revived, with such disastrous consequences for philosphy, by Descartes, has been fighting a losing battle with the demands of discursive reason for verification and experiment. It is becoming increasingly difficult to claim finality for propositions about matters of fact, solely on the ground that they are intuitively evident. The disconcerting frequency with which changes in physical theory have taken place, the discovery of alternative sets of principles to replace those maintained most dogmatically, has taught many thinkers some modesty in their pretensions to final statements.

The development of mathematics in the second half of the last Century was successful in showing up as unwarranted the doctrine that a sensory, spatial, intuition was necessary for the pursuit of the science as a deductive one. Carefully formulated rules of procedure replaced uncontrolled intuitive methods, so that the validity of the theorems was made to depend upon the conformity of their derivation to those rules, and not upon an appeal to a vague self-evidence. The fruitful character of the rigor thus introduced is well known.[1] Nevertheless, although sensory intuition is no longer regarded as a method of proof, there is still a striking insistence in most schools of mathematicians who have reflected upon their science, that intuition in some form is indispensable for its foundation. The locus and nature of the indubitably known data has been altered, but the essence of the doctrine of self-evident truths remains. We turn to a brief survey of some recent theories on the foundations of mathematics.

The practical origin and import of mathematics in facilitating certain physical activities is a matter of record. Measurement and calculation, as has been shown above, contribute to the accuracy and the definiteness with which certain operations upon qualities can be carried out. The symbolic statement, for purposes of communication, of the numerical relations between segments of existence, and the symbolical elaboration of the methods which are successful in controlling and predicting the behavior of bodies, is from the point of view of existential meaning, the proper task of numerical science. From this point of view, mathematics is the study of those operations in existence which the physical interconnection of things makes possible. There is, however, a wide gulf between the primitive operations with physical objects which started the career of mathematical science, and the highly abstract science we know today. Mathe-

[1]The complete success of the logistic movement has not yet been attained. Even if one neglects the protests of men like Holder, Poincaré, and Hilbert, that there is an irreducible alogical moment present in their science, the well-known difficulties with the multiplicative axiom or the axiom of reducibility must be a warning to the over-confident.

matics has become a deductive science, and in becoming that, it has apparently divorced itself, its refined and subtle intellectual analyses, from the crude, approximate, measuring operations of common day activity. The theorems of mathematics, it has developed, are not categorically true of existences. The status of the theorems is hypothetical, and they are theorems about existence only in so far as the axioms from which they are deduced are exemplified in existence. Moreover, the connection betwen axioms and theorems is apparently far removed from the kind of necessity which the brute compulsion of existence exercises. Consequently, even if the history of mathematics were to show a constant process of generalization of those primitive operations with things, it must seem a paradoxical demand that the ultimate (in the sense of most assured) bases of mathematics should be sought in the adequate symbolic representations of well-attested physical operations.

Nevertheless, however hypothetical the existential meaning of mathematical theorems may be, the connection between premise and conclusion can not be a hypothetical connection; the validation of mathematics as a deductive system is not complete until the principles of inference are validated. It is with the certification of the general principles of mathematical procedure that recent controversies are concerned. The bold, but on the whole successful, efforts on the part of mathematicians and logicians, to discover a single set of consistent principles from which the vast body of mathematical doctrines could be deduced, has terminated at some points in "antinomies," such as the well known paradox arising from the concept of the class of classes which are not members of themselves. From the further efforts to circumvent such antinomies and to guard against new ones, at least three different views as to nature and basis of mathematics have become current.

The first view, best known to English readers, is the theory of Frege and Russell that mathematics is identical with logic, since all the specifically mathematical theorems are deducible from the primitive ideas and propositions of logic According to this theory, no *special* mathematical or physical intuition is required for establishing any of the propositions of abstract mathematics, and this science, therefore, has as its province a realm of platonic forms significant and explorable without requiring any reference to the operations of matter. This theory is not now held in all its purity even by Russell, since, as has been noted, certain axioms must still be assumed, without being derivable from the purely logical principles, as pragmatically necessary for the development of large portions of mathematics.

A second view is championed by Brouwer and Weyl, and receives some support from the later work of Husserl and his school. Mathematics, according to this philosophy, must be based not upon logical principles, but upon a "pure intuition" of the integral numbers involved in the intuition of time. The *linguistic formulation* of mathematics, it is claimed, is not of the essence of its subject, and hence the logic, which applies only to this formulation, can not be carried over to certain portions of the proper subject matter of mathematics. Traditionally, the criterion of the "existence" of mathematical entities has been freedom from self-contradiction, and in many "existence theorems" no clue is given how to *construct* or *evaluate* the entities whose definition is shown to be consistent with the body of mathematics. In the theory of aggregates, for example, it has

been shown that, on the assumption of the multiplicative axiom, every class may be "well-ordered," and that in particular the continuum may be so ordered; but no indication is given *how* the "well-ordering" is to be effected. But Brouwer and his followers take as their point of departure a conception of "mathematical existence" which makes the production or *intellectual construction* of the defined entities essential for their existence; existence theorems which prove *consistency* but which do not indicate how the evaluation or constructions are to be made, are held to be *meaningless.*

It is the insistence by Brouwer upon the necessity of a pure intuition of the natural numbers, in which all the requisite logical and arithmetical relations are also revealed, that evokes sympathetic support for his doctrines from the phenomenological school. The cardinal principal of the phenomenological method has been stated by one of its practitioners in the following terms: "Every genuine phenomenon must reveal its own essence to him who apprehends it adequately in an immediate, intuitive grasp."[2] Such an intuition lays hold, with an indubitable certainty, of those entities (and only those) which are declared to be immanent within a "pure consciousness." This consciousness is vaguely defined as a type of ideal continuity present to all conscious beings and to which matters of sense are transcendent. Logic is therefore affirmed to be the science of *a priori* forms known independently of the empirical sciences, but competent nevertheless to legislate and evaluate their methods. A new "eidetic science," — a science of essence — is therefore declared as possible; each essence must be known intuitively independently of other essences, so that the eidetic science turns out to be a kind of positivism transplanted from the realm of sense to that of intelligibles. Indeed, Husserl is willing to call himself a positivist, if among the indubitable simples there be included such essences as "$a+1=1+a$," or "a proposition is not colored." The heart of the doctrine is that only what is "this-side" of consciousness is absolutely certain; and that of empirical things we can not have that luminous knowledge which is beyond every doubt. It is not surprising that for some members of the school, empirical uniformities are fundamentally intelligible.[3]

The third view as to the nature of mathematics is advanced by the "formalist" school of Hilbert, Bernays, and Ackermann. Hilbert also denies that mathematics can be based upon logical principles alone, and demands therefore the intuition of certain extra-logical, concrete objects. All doubt as to the consistent nature of mathematics will be removed, it is alleged, if such objects are capable of being surveyed once for all, and if the relations in which these objects stand to one another is immediately revealed in the very intuition of the objects themselves. Those objects, however, are not the natural *numbers* of Brouwer; they are the physical *signs, marks,* or *numerals,* with which the

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[2]Becker, *Mathematische Existenz,* p. 309. See also Husserl, *Jahrbuch f. Philosophie u. phanom. Forschung,* Bd. I p. 43.

[3]"The relations between essences are unseverable, they are prior to empirical laws. They constitute the fixed limits within which empirical laws may develop, but they can never be superceded by these laws. Essential relations constitute that which is intelligible in any occurrence. Mere empirical laws are fundamentally unintelligible, even if we have accustomed ourselves to them so that they no longer seem strange, and are regarded as inherently understandable; properly speaking, they are never that." Becker, in *Jahrhuch f. Philos. u. Phanom. Forschung,* Bd 6, p. 400. Husserl's own detailed exposition of his method is found in the first volume of the *Logische Untersuchurgen,* as well as in the initial volume of the *Jahrbuch.*

mathematician operates. Consequently, it is Hilbert's program to reduce the whole of mathematics to a formal, *meaningless* play with marks, so that the pursuit of mathematics becomes a glorified game of chess. The *rules* of the game, designated as "metamathematics," have a meaning, but only in the terms of the meaningless operations with signs to which "mathematics" has been reduced. It is Hilbert's aim to show that "mathematics" so formalized, is constituted by operations which never mutually destroy or check each other, and that consequently *traditional mathematics* is thereby exhibited as self consistent.[4]

Now, whether mathematics is made to rest upon a pure intuition of the integers, or whether its foundation is discovered in some immediate knowledge of concrete marks, or whether we assert "luminously self-evident" logical principles; in every case the appeal to self-evidence or immediate knowledge is a begging of the question, if that self-evidence is taken as a *guarantee* that the intuited data are both *adequate,* to support the mathematical structure, and *self-consistent,* to permit its further development. Mathematics is more than the structure of its *symbolism,* for example, and if the latter is truly to symbolize, the reference and adequacy of sign to signified matter must be carefully and laboriously worked out, not taken for granted as self-evident. If we remember how sad has been the history of philosophies and sciences which have worked on the premise that self-evidence is something with which we *begin,* we may hesitate to endorse an attempted foundation of any discipline upon such a basis even if no alternative premises are known to be possible. The price that has been usually paid for an intuitive foundation of knowledge has been high. It has been the sharp separation, on the one hand of physical and mental activities which involve trial and error, and demand constant verification as the conditions of certainty and consistency; from the immediate grasp of first principles on the other. A philosophy which sanctions the *a priori* in its usual forms, leaves the connection between material and formal logic unexplained, and too often makes a confusing puzzle of both the applicability and the application of logic to existence. That puzzle becomes a nightmare if the intuited data are declared to be metaphysical simples, whether atoms of sensation, atomic propositions, or simple ideas, each self-contained, each a diminutive absolute; it is inevitable that on such views belief in an "external world" or in a casual nexus should be nothing less than superstition.[5]

Theories of intuitive knowledge must inevitably face the difficult question: What is the evidence for self-evident principles? What are the guarantees that alleged self-evidence and certainty are truly such? Two types of argument, a psychological and logical variety, are generally offered to substantiate the belief in self-evident truths.

The psychological argument, somewhat crudely stated, is this: With respect to some propositions we have a *feeling* of affirmation which makes belief in those propositions inevitable, and the intensity of our feeling may be taken as a sign of verdicity of the proposition.[6] It is probably sufficient to retort that, like Peter's idea of Paul, the strength of our feelings is evidence more for the

[4]These conflicting views are clearly presented in Fraenkel, *Zehn Vorlesungen uber d. Grundlegung der Mengenlehr,* and in his *Einleitung in die Mengenlehre,* 3 ed. See also Becker, *Mathematische Existenz,* and Weyl, *Philos. d. Mathem. u. Naturwiss.*

[5]Wittgenstein, *Tractatus Logico—Philosophicus,* p. 109.

[6]*Cf.* e.g. Höfler, *Logik,* 2 ed. p. 60.

habits of the inquirer than a guide to connection of ideas or things. Dogmatism based upon the inconceivability of the opposite, or upon alleged apprehension of clear ideas, must too often swallow its own pretensions. Since the discovery of geometries alternative to that of Euclid, for example, we can hardly claim self-evidence for our supposed sensory intuition of spatial relations; and all the Nelsons who continue those claims reveal themselves as pathetic dictators to the tides of scientific progress. Even Husserl has introduced important corrections into his phenomenological analyses.[7] Moreover, the affirmation that a proposition *is* evident, is *another* proposition which requires certification; so that on pain of an endless regress or of a circular argument, the mere presence of such psychological feelings of conviction is not equivalent to a *knowledge* of the self evident character of the proposition in question. Indeed, the sanest logicians who operate on the basis of "evidence" as a criterion of truth, deny infallibility to any *single* act of perception or judgment; "evidence" for them is not an immediate psychological datum, it is rather the chain of arguments which support the proposition under judgment.[8]

The logical argument is very simple, but powerful: Any reason we may offer for a statement may itself be challenged, and we are thus compelled, on pain of an infinite regress, to acknowledge some general principle immediately grasped. If there is no *first* term in the series of mediate judgments which constitute our knowledge, at which we can stop to say "This is so and so," then *nowhere* in the series is such an affirmation possible.[9] Nevertheless, the force of this argument is considerably reduced when the following points are noted. (1) The logical priority of certain principles must not be supposed to indicate their *temporal* antecedence. That which may seem to us as luminously clear and adequate to support an argument is not *therefore* intuitively known. The axioms of Euclid, or the laws of moving bodies postulated by Newton, are well known to be subsequent in the order of our knowledge, to the theorems in geometry and mechanics to which they lead. Moreover, the grounds for our belief in such principles are found not in any character of self-evidence they inherently possess; they may be more certain than any one of the facts they help explain only because of the cumulative evidence from *all* the facts they so explain. (2) The argument that there must be a first term in a series of mediate judgments, misapprehends, as Peirce has shown, the nature of a continuous series.[10] A sequence of judgments may be continuous, and there may be a time in the past such that no judgment belonging to that sequence was then made; but, nevertheless, there need be no *first* in that series of judgments. "Cognition arises by a *process* of beginning, as any other change comes to pass." (3) The difference between the immediate quality of our thought as an *event*, and its relational or inferential character as *knowledge*, must be recognized. The distinction so noted is precisely the distinction of which any sign is capable: it is a sign *in* some respect or quality, and has an individualized existence; and it is a sign *for* some other events, which

[7]See Nelson, "Bemerkungen über d. nichteuk. Geometrie," in *Abandl d. Fries-chen Schule*, N.F. Bd.I and Hartmann, *Metaphysik der Erkenntnis*, 2 ed. p. 488 ff.

[8]For example, Geyser, *Auf dem Kampffelde der Logik*, p. 230; Coffey, *Epistemology*, Vol. II p. 274.

[9]Cf. e.g. Russell, *Problems of Philosophy*, p. 175; Husserl, *Logische Untersuchungen*, Vol. I p. 85.

[10]*Journal of Speculative Philosophy*, Vol. II, p. 112. See also, H. B. Smith, "Postulates of Empirical Thought," *Journal of Philosophy*, Vol. XXV, p. 319.

are not identical with it. The immediacy which accompanies our thought as a temporal process, can not therefore be identified with, or taken as a guarantee of, the *logical* function of the process.

But whatever be the force of the arguments for intuitive knowledge, an alternative doctrine may be formulated.[11] Just as existence is prior to any distinctions that are made in it, so the unreflective activity of an organism is temporally prior to its taking thought; consequently, adjustments to the environment may be made without a knowledge of the principles that are involved. Such adjustments are tentative, subject to error, and the formulation of the principles that are involved is quite as experimental as are the physical adjustments. Rationality is therefore a late development in the career of an organism, and the activity of the organism is fixed both by its own habits as well as by the habits of the environing world. When those habits become formulated as principles, the characters of objects may be restated, re-arranged, and redefined, not in the order of our observation of them, but in terms of such principles as will permit the inclusion of some characters in others. Conclusions are determined by premises in accordance with general habits of thought, which have emerged as successful in producing reliable knowledge. The general principles of logic, considered as habits of thought, express and reflect the habits of that which is not changeable by thought, but which exercises compulsion upon thought. Consequently, these general rules or habits are not capable of being assumed, or put on and off, at will; they are not certified by an intuition. They are rather the residue of a sifting process in which *operation* with those principles, carried on for the securing of knowledge that must on occasions stand the test of action in the physical world, is the only guarantee we possess of their efficacy and compatibility. It is therefore not impossible that some of our most cherished rules may yet break down, and that the most rigorous proof yet devised may turn out to be leaky.[12] If we are more certain of the laws of logic than of any other principles, it is not because they reveal themselves in an epiphany, but because all our operations upon things, all our discourse and communication, gives them credence and support. "The controlling force in reasoning is not reason, but instinct and circumstance," remarks Santayana. It was an excellent Hegelian insight which, recognizing the dual roles of mediacy and immediacy in knowledge, found ready-made solutions and technical expertness the end products of long reflection, however familiarly or immediately they may present themselves. "Self-evidence" is a character that belongs to propositions at the *end,* not at the beginning, of the knowledge process.

It may be urged that a vicious circle is involved when an attempt is made to validate general laws of logic, since they must be used in that very process. There is, of course, no possibility of demonstrating the most general principle of demonstration. May not one ask, however, whether the binomial theorem is true by considering definite values of the exponent? All that is really at issue

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[11]The source for this alternative doctrine is, of course, Peirce. See *Journal of Speculative Philosophy,* Vol. II; *American Journal of Mathem.* Vol. III; *Chance, Love and Logic,* ed. by M. R. Cohen, Part I; also articles in Baldwin's *Dictionary.*

[12]The history of mathematics testifies to the fact that not even the domain of the queen of the sciences is exempt from error. There is a vigorous endorsement of the view here taken in Bôcher, "The Fundamental Conceptions and Methods of Mathematics," in the *Congress of Arts and Sciences,* Vol. I, p. 459. *Cf.* also N. Wiener, "Is Mathematical Certainty Absolute?" *Journal of Philosophy,* Vol. XII, p. 568.

is whether special instances may be used to win conviction and support for a general principle. Consequently one is not involved in any impossible scepticism when one asks whether the formulated principles of a demonstration are, as a matter of fact, the principles actually involved in a demonstration. To think is as much a natural event as to eat, and one may do either without using consciously the general principles implied. One may ask significantly what are the conditions for valid inference, even though in raising the problem one is, it is hoped, thinking validly. It is only Scholasticus who would enter the water only after he has learned how to swim.

This rejection of intuition as a certification of truth, whether of things or principles, has its moral for some of the issues at stake in a philosophy of mathematics and logic. The desideratum in mathematics is that the process of deduction should never lead to contradictions. The classical method of demonstrating the self-consistency of any branch of the science is to show that it is *as* consistent as some other branch, (usually arithmetic or logic) by exhibiting numbers and arithmetic relations, for example, which will "satisfy" the mooted axioms. But is arithmetic itself self-consistent? The answer usually given has been an appeal, to an intuition of intergers, or to constructs from logical simples intuitively founded, or to empirical objects "seen" to satisfy them. But, really, appeal to intuition is question-begging, whether that intuition consists in an exhibition of logical constructs or empirical objects, *unless* these are already verified as capable of bearing the weight thus thrust upon them. In every case when entities are exhibited as satisfying certain theorems, there is a prior question which must be settled: Do these entities have the character which we suppose them to have? Are they of a nature so that such-and-such properties may be significantly attributed to them? When it is said that no physical object is self-contradictory, the statement may mean either, that consistency is an attribute of things in a logical system (and therefore objects taken as brute existences are neither consistent nor self-contradictory); or, that when a thoroughgoing analysis of objects is carried through, self-contradictory attributes never appear in the same context. There is no inconsistency in the act of looking for Venus at midnight. But if it is asserted that Venus is an evening star, it is inconsistent with that proposition to look for it in the morning. Questions as to the consistency of events and actions can arise only in so far as they are declared to be imbedded in a relational network. Consequently, the first interpretation of the statement concerning the non-contradictory nature of physical objects can be of no value in proving the consistency of propositions by the exhibition of objects, as such, which shows the axioms of a system to be consistent. It is the extended operation with objects and with the axioms in various contexts that accomplishes this.[13] Actions, objects, and propositions must be first taken into a system of meanings before they can become contradictory or consistent.

What guarantees are there, therefore, that the axioms one has adopted do not involve contradictory theorems? It is the point of this section that there are no absolute guarantees. If we feel more certain about some part of our knowledge than of the other parts, it is because it has stood the test of centuries of

[13]In this respect, no priority can be given to physical objects over ideas or logical constructs. The demand that empirical objects be the ultimate exhibited elements is, moreover, a program and not an accomplishment. How is one to demonstrate the consistency of the rules of the differential calculus by exhibition of empirical objects?

use. The recent discoveries of "antinomies" in mathematics should make us cautious about the unrestricted application of so-called universal principles. The history of logic is witness to the development of our knowledge of logical doctrines—the logic of relations is a late growth as an explicit formulation; and the new formulation of valid leading principles e.g. theory of types, is a slow process.[14]

Hilbert's heroic attempt to find criteria of absolute consistency, even if his program can be completed,[15] has not quite the significance he seems to attribute to it. Not much is won by calling "metamathematical" those principles which are in dispute. Moreover, however indispensable symbols may be as safeguards and resting places for inference, it is not themselves they symbolize. Hilbert's characterization of mathematics as a play with meaningless marks according to fixed rules, can not be an adequate statement of what mathematics is. For even as pawns in a game the marks are not *merely* marks or physical things, since each represents the class of all marks in some sense like itself. Even on such a crude behavioristic theory of mathematics Hilbert's marks have a symbolic role. One may indeed take certain elementary combinations of signs as ultimate data, but only in the sense that one takes scale readings or sense data to be ultimate discriminated elements in a science. It must not be forgotten, however, that there is a long process of inference involved in reducing a complex problem to such easily manipulable and observable items. If those inferences are taken for granted, it is only because of their efficacy in other contexts.

In the denying the claims of intellectual intuition to be the tribunal of truth, one is not compelled to embrace a sensationalistic empiricism which identifies thought with sense perception. Thought moves on a plane different from the clash of atoms or the stimulation of nerve endings. But at least to one reader of the *Theaetetus,* if Plato's refutation of sensationalism masquerading as knowledge is complete, his arguments are equally fatal to a positivism of essence.

II

Preoccupation with methodology has often led to the denial of an intimate connection between method used and the irreducible traits of subject matter explored. But can a method ever be purely a method, and reveal nothing about the subject matter within which it operates? Can characters of a subject matter ever be formulated, without the occurrence of such a formulation indicating something final about that subject matter? Can the fact that generic characters of existence become objects of knowledge only as *conclusions* of an inferential process; and that objects of knowledge represent the transformations of existence given to a non-cognitive experience, produced by the thinking process for purposes of control and continuity; can that fact be used to discredit those characters as generic characters of existence?

Objects of *knowledge,* it can be granted, possess some traits which belong to them in virtue of the reflective context within which they occur, and such traits

[14] *Cf.* Rougier, *Paralogismes du Rationalisme,* p. 441 ff.

[15] Cf. the remarks of Fraenkel, *Einleitung in die Mengenlehre,* p. 377ff. Professor Sheffer is in possession of an absolute proof for the consistency of a set of axioms. But the few hints he has given of it in his published writings, have not been sufficient for me to understand its method.

cannot be attributed directly (that is, outside of this context) to an existence not involved in such a reflective process. These specifically logical traits can be recognized as such, presumably, by comparing the traits of objects of knowledge with the traits of objects given in a direct, non-reflective experience. Nevertheless, if the nature of existence is discovered not by some prior definition of what it is; if it is discovered by an experimental inquiry; the outcome of that inquiry must in some sense be identified with features which existence possesses even when not involved in such inquiry. When light falls on a metal plate and an electric current flows in consequence, the relation between intensity or frequency of light and strength of current, which is established by an inquiry, is an independent factor in the domain surveyed, even though the quantitative manner in which that relation is stated and established is not.

Consequently, if the traits of objects found in an inferential process are *merely* logical traits, then no ontological characters of objects can be discovered *within* a reflective inquiry. But if metaphysical traits are not found within a reflective context, the only occasion when they could be noted would be in those non-reflective experiences of enjoying and possessing to which some recent philosophers have so eloquently called attention. If, however, ontological traits do not emerge in a reflective inquiry, we cannot properly be said to *know* them, even though we may experience them in other ways. Over and above the logical traits of objects of knowledge, ontological characters must be present as well, if the generic traits of existence are to be *known* and not only *possessed* or *enjoyed.* One does not ask a significant question about the connection of large and metaphysics, therefore, in asking whether metaphysical traits are known to be something *additional* to the logical traits; one must ask whether ontological traits can be identified, at least in part, with logical traits.

One must therefore ask on what grounds metaphysical characters can ever be predicated. How do we *know,* for example, that specificity, interaction, and change, characterize all existence, and that these distinctions are not merely logical, made for purposes of getting along in this world, but are characters of an independent existence? Why, moreover, do we impute the features presented in human experience to a nature embracing, but containing more than, that experience? But such questions inevitably lead on to the conclusion that if logical traits are cut off from ontological traits, so that the former have no prototype in the latter, the belief that traits like those mentioned are exhibited not only in the human foreground but are outstanding features of nature throughout, is itself untenable. For the single argument which can be advanced against those critics who deny to nature, independent of experience, the characters disclosed in human experience, is one drawn from the principle of continuity. The considerations which lead to the adoption of the principle are not relevant here; it is the implication of its adoption which must be traced. But if it is adopted, then we must concur with the conclusions of Professor Dewey: "One who believes in continuity may argue that, since human experience exhibits such traits nature *must* contain their prototypes." "For this reason, there are in nature both foregrounds and backgrounds, heres and theres, centers and perspectives, foci and margins."[16] On this argument, therefore, it seems necessary

[16] *Journal of Philosophy,* Vol. XXIV, p. 58.

to suppose that while there may be other ontological traits besides the logical, nature must contain the prototype of the logical as well.

There is no way of comparing the traits of nature manifested in the human scene with traits of nature existing in that background which is not in that scene. There is no way of discovering, by using reflection, that nature does not possess the characters she is discovered to possess *in* reflection. If, to use a figure of Professor Dewey, objects of knowledge are the finished tools obtained by transforming crude ore, then it is the finished tools which are known. That nature does possess non-logical characters is a fact to which our common-day experience testifies. Such characters are real; but not more real than the characters which reflection discovers and notes. Their ontological status is the same. One has as little reason to expect to discover logical characters outside of an intellectual or knowledge situation, as one would have to expect to discover a sensible quality in a non-sensory way.

To one who is committed to a whole hearted naturalism, the continuity between logic and metaphysics can not be broken. The logical method we use, as has been suggested, is a method developed in the successful operation upon the things of existence—things, which in their physical dimension, exercise brute compulsion upon us, and which are discovered to stand to each other in relations approachable by intellect. By the very principle of continuity, the method must be more than method, even if the function of that method is the human one of making more secure the eventuation and enjoyment of immediate qualities. The method must in some sense reflect or refract orders of things which are knowable orders; orders which are never completely known, but which become known only if the existence given non-reflectively is transformed and modified. For we know things in the sense that we know the *relations* between them. To know the relations we must manipulate between them. To know the relations we must manipulate and change the things. Is there any reason to believe that in getting knowledge we ever determine and fix what those relations must be?

One need not believe that it is the inherent properties of the ultimately real that are disclosed in knowledge; one may agree that it is the correlations of changes which is the goal of inquiry, and that such correlations, when stated, are hypothetical, to be tested by consequences rather than by antecedents. One may say that the objects of science are the objects of the thought of reality, from the point of view of the most highly generalized aspect of nature as a system of interconnected changes; one may insist that the sole meaning of the relations thought of, is to be found in all the *possible* experienceable consequences which they entail. But it nevertheless remains true that such relations are discovered as integral factors in nature, and that however hypothetical some of them may be when generalized for wide regions, in some cases no alternatives to our believing in them are obtainable. Principles which every one must have who knows anything about existence, Aristotle remarked, are not hypotheses. His remark finds an echo in the statement of Professor Dewey that "nature has both an irreducible brute unique 'itselfness' in everything which exists and also a connection of each thing (which is just what *it* is) with other things such that without them it 'can neither be nor be conceived'."[17]

[17] *Loc. cit.* p. 63.

Nevertheless, it is easier to claim that there must be some connection between logic and ontology, than to exhibit that connection in detail. And indeed, not more than a gesture in that direction is attempted here. But first certain reservations must be made.

It is granted that a non-ontological interpretation of logic can be sustained, by confining the distinctly logical to the symbolic or psychologic mechanism involved in getting things thought and said. If the "laws of thought" are taken to define the use of words like "is" and "not," or to describe supposed facts of psychology, then obviously they do not refer to generic traits of existence. The only contention that is here advanced, is that such interpretations do not do full justice to the operative role of the laws, and fail to look beyond the words to the occasions which fix their use.

Secondly, the laws may be understood as methodological resolutions we make to ourselves in conducting inquiries. Just as the choice of geometry is a matter of some convention and cannot be proved false by experiment, so the laws of logic, it is claimed, are our postulates for organizing our experience, are the rules of our method and not laws of nature. But one can very well agree with what this theory affirms, without affirming what the theory denies. The rules may very well be rules of method, but nevertheless the method must be congruous with the realized behavior and the unrealized potencies which existence exhibits. Moreover, the laws of logic are unique in that they turn up in every conceivable inquiry, and in that no alternative postulates have ever been successfully applied. One suspects, therefore, that they represent factors invariant in *every* subject matter.

Finally, it is not denied that logic is concerned primarily with the ways of getting knowledge, with our second intentions of things, with things as objects of reflective thought. Logical principles are concerned with the mutual relations of propositions, not with the mutual relations of things: and propositions bear to each other relations incomparable in some ways with the relations between things. Consequently, a fundamental meaning which must be assigned to the principle of contradiction, is that a proposition can not be both true and false; and to the principle of excluded middle, that a proposition must be either true or false.

Nevertheless, from a reading of these principles which make them declare something about propositions, we are led to metaphysical readings: Uniquely determined subjects can not both have and not have the same character; existence is determinately what it is; determinate existence excludes certain characters, and a determinate subject must either possess or not possess a given attribute; the characters which things possess fix the relations between propositions about them. It is true that propositions, and not things, contradict each other. But the ground of contradiction is to be found not in the propositions, but in the nature of things which the propositions say. The saying of things is not the things said; and yet the saying conveys what the things are. If the principle of excluded middle does not tell which of two attributes or relations a thing does have, it does tell us that, given an attribute or relation, a thing either has or has not got it.

It is therefore perfectly true that when things become involved in inference, they assume logical traits which they did not possess outside of such a process.

Spectrum lines, for example, exist as physical things, and are the physical effects of other events; they are physically connected with these other events so that they may become signs of them in virtue of that connection. But they do not function *as signs* outside of an inferential situation. When things are regarded as merely existing, they interact with, push, pull, beget, one another; as existing, things and their behavior have an immediacy, a qualitative uniqueness and individuality, which is underived. Nevertheless, the connections between individualized behaviors can be grasped intellectually and stated in general terms, so that the generalized relations are illustrated by the individualized behaviors, although their meaning is not exhausted in any one behavior. A body falls to the earth. We find, that, on taking note of the distances it traversed in its fall and of the time lengths it required for them, a definite relation can be stated not only between the distances and and times *already* completed but between the future distances and times the body will occupy. The *isolation* of distances and times is an act of thought, and outside of the reflective art, distances and times do not exist isolated from the other qualities a body has. In this sense, the proportionality of the distance a body falls to the square of the time it requires to do so, is a character of the fall of the body only *within* the knowledge context. But if that proportionality expresses a law of nature, it is because distances and times are isol*able* though not isol*ated* characters of things; and because things stand in relations with one another which are statable in terms of relations between isolable characters. Discourse necessarily particularizes a relation in trying to convey it. There is therefore both more and less on a map than in the country-side. The lines of the map have no counterpart in the country-side *as* lines; but they denote relations between things not found on the map.

It may be objected that in limiting the discussion to determinate being, the question of the relation between logic and ontology has been begged at the outset. Logical distinctions are applicable to determinate subject matter only. But in admitting the truth of that proposition, it must also be admitted that in so far as there is *complete* indetermination or chaos, *nothing* can be said. Chance is a metaphysical character, while at the same time absolute chaos or pure contingency is as much a limiting concept as pure form. Pure matter and pure form, each equally unthinkable, are the two limits within which nature operates. Even the contingent, consequently possesses an aspect of the determinate, just as the existentially determinate possesses an aspect of the precarious.

But is determinateness, it may be asked, ever found as a "direct" property of things, or is it discoverable only in non-existential subject matter? It is sometimes believed that the existence of change or transition indicates that all determination is dialectical and non-existential.[18] When it is said that an existing arrow must be either at-rest or not-at-rest, it is urged in opposition that the arrow may be rest*ing* or mov*ing*, and that no existent arrow is ever completely at rest. "Rest" and "motion" are therefore ideal in nature, the opposition continues, setting *limits* toward which one may work in existential affairs. But one may well ask whether this dilemma is not due to a failure to define operationally and denotatively the idea used. "Rest" may indeed be a universal whose instances are qualitatively dissimilar, so that the meaning of "rest" in different

[18] *Cf.* Schiller, *Formal Logic*, p. 117.

contexts is different. But fix the context, make explicit by pointing to a set of operations or conditions what it is to be at rest, and the arrow meets or does not meet these conditions. It is because "rest" is first conceived in terms of an unexpressed ideal, rather than in terms of empirical procedure, that the apparent inapplicability of the excluded middle is plausible. (It is perhaps needless to point out, moreover, that the dialectic invoked against an ontological logic ought to be equally fatal to the disjunction, that the arrow is either in transition or not in transition.) Two events are said to be simultaneous not intrinsically, but by reference to the explicit operations which define simultaneity. Two events may be judged differently with respect to simultaneity, as an arrow may be judged differently as to its being at rest—but not when the characters defined are first fixed operationally. If "arrow" can be defined existentially and not in terms of intrinsic characters, may not "rest" be defined in the same unambiguous way?

However much consistency of thought is conditioned by the nature of things, nevertheless it is only thinking that can be said to be self-consistent. Thinking is a selective process, and the principle of excluded middle, for example, is applicable only if we discriminate relations. It is sometimes concluded, therefore, that relations are being hypostatized into independent existence if we suppose logical principles to be directly applicable to existence. But does it follow that because a universal does not exist *by itself*, it does exist *at all*, apart from the abstractive process? Does it follow, because isolation and abstraction are distinctive intellectual operations, that the objects of thought are not intrinsic characters of nature? Must one deny, because to be effective thought must approach nature piecemeal, that nature inherently has heres and nows, centers and perspectives? The systematic character which some things possess is reflected in our discourse and thought in a way which includes the accent of our speech and habits of our thought and tradition; but it is not that accent and the habit which endow things with system.

On the ground that the future is incomplete and our knowledge of it is vague, it is sometimes asserted that logic breaks down in trying to deal with it; a proposition is not either true or false when its subject matter is contingent, since the proposition may be undetermined in its truth value. Recently, the more drastic claim has been made by Becker that certain series of integers possess this same contingent character.[19] Consider the series of numbers of the form $X = 2^n + 1$ for increasing values of n, and define another series K such that if X is prime the number 1 is assigned to K; if composite, the number 2. The series K then has this appearance: 1,1,2,1,2, . . . If we ask whether, after the sixteenth place, the number 1 is contained in K, Becker maintains that we not only can not answer either "yer" or "no"; we can not even suppose that the possible answer is exhausted in these alternatives, we can not even say "yes or no." His reason is that our knowledge of primes of this form does not extend beyond $n=16$, and therefore the occurrance of 1 in K is contingent upon our determining the prime or composite character of X when n is greater than 16. K is thus a "growing", contingent, series with respect to the property of containing 1; its determination lies in the future.

[19] *Mathematische Existenz*, pp. 54 ff, 307 ff.

Those who understand by the contingency of nature something else than human ignorance must admit that the future is not completely fixed at any time prior to it. It is admitted therefore that propositions about the future are not categorically true or false. The proposition "In 1933 Hoover will be the President of the United States," is a proposition of this kind. Nevertheless, even an "incomplete whole" may be the subject of prediction, on the assumption of certain permanences throughout the totality whenever completed. Conseqeuntly, the proposition "In 1933 Hoover will either be President of the United States, or he will not be President" is true, *if* Hoover is still alive in 1933 and *if* the condition of the world at that time will be such that to be President or not will still be significant. Paraphrasing Aristotle we may say, "Hoover must either be President in 1933, or not, but it is not necessary that he should be President in 1933, neither is it necessary that he should not be, yet it is necessary that he either should or should not be President in 1933." It is possible, therefore, to enumerate exhaustive formal properties about the future, if that future is not altogether independent of the character of the present, or of any particular time. Predictions about the future, as well as about the past and present, require that some relations which are and have been exemplified in existence will continue to be significant for it. In general, this is a hypothesis which in particular cases is often proved wrong. But were it always wrong, it would be difficult to see how the future can be the subject for discourse at all, or how the present, when it has become the past of such a future, can be thought and investigated. Hence a future completely different from the present is as little intelligible as a past so dissimilar; no prediction about it would have any value. But in entertaining significant alternatives about the future, alternatives for which the excluded middle holds, the essential contingency of future occurrences is not denied.

In spite of the indeterminism which Becker claims for his "growing" series, he admits that such series are fully determinate with respect to certain properties.[20] The above series of numbers X is determinate with respect to the property "congruent 1 mod 4." What is the nature of the contingency which Becker affirms of the series K?

To locate indetermination in a number series is not a happy device, and is inevitably confusing. For the imputation of indeterminism to the future means not only (if it means that at all) that our knowledge is incomplete; it means also that not all the present possibilities of events can become actual, and that the actualization of present tendencies of events awaits an indefinite number of contingent factors. The indeterminateness which may be attributed to number series can hardly have the same sense as this. For if we begin a series S arbitrarily thus: 1,5,2 . . ., then any proposition made about the numerals not yet written down is indeterminate in some respects, though not in all; the terms of the sequence will be, at least numerals, although we do not know in what order the individual numerals will appear. To be *numerals,* the terms must possess certain characters which are independent of the order in which the numerals appear in this particular sequence. However, in the series K, even this indetermination disappears. The occurrence of a 1 is no longer arbitrary,

[20] *Loc. cit.* p. 9.

but is completely determined by the prime or composite character of X, even though the chain of determination between the form of X and those characters may be altogether unobvious. The physical operation of *writing down* 1 or 2 is, of course, dependent on our knowledge or ignorance; it is also dependent on many other factors. Since the contingency due to our knowledge in this case would vanish if the theory of numbers were complete, on Becker's view, the contingency of a subject matter is identical with the partial character of our knowledge. The danger is very great, therefore, that if with him we take series such as K as models of existential changes, the characteristic incompleteness of the flux of events will be overlooked, and that the pervasive contingency of existence will be regarded as a merely human attribute.

Vita

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